Contents

Nomenclature xv

Notation xvii

1. Introduction 1
- 1.1. Motivation . . . 1
- 1.2. Related work . . . 4
  - 1.2.1. Embedded method . . . 4
  - 1.2.2. Partitioned coupling . . . 5
- 1.3. Aim of this work . . . 7
- 1.4. Outline . . . 7

2. Fluid dynamic equations 9
- 2.1. Compressible viscid Navier-Stokes equations . . . 9
  - 2.1.1. Boundary conditions . . . 11
- 2.2. Compressible inviscid Euler equations . . . 12
  - 2.2.1. Boundary conditions . . . 12
- 2.3. Linearized Euler equations . . . 13
  - 2.3.1. Boundary conditions . . . 13

3. Discretization in space and time 15
- 3.1. Spatial discretization - Discontinuous Galerkin method . . . 16
  - 3.1.1. Compressible inviscid Euler equations . . . 16
  - 3.1.2. Compressible viscous Navier-Stokes equations . . . 19
- 3.2. Explicit time discretization - Runge-Kutta method . . . 23
- 3.3. Numerical oscillations - Gibbs oscillation . . . 24

4. Numerical Framework 29
- 4.1. *APES* simulation framework . . . 29
  - 4.1.1. High-order Discontinuous Galerkin solver - *Ateles* . . . 31
  - 4.1.2. Integrated coupling approach - *APESmate* . . . 32
- 4.2. External coupling approach - *preCICE* . . . 33

5. Embedded boundary method 35
- 5.1. State of the art - Geometry representation . . . 36

5.2. Volume penalization method - Brinkman penalization . . . 38
5.3. Representation of the masking function χ in high-order Discontinuous Galerkin 41
5.4. Evaluation of the masking function χ in the numerical scheme 43
5.5. Over-integration - Cost estimation 50
5.6. Specification of the masking function χ 52
5.7. Implicit-mixed-explicit Runge-Kutta method 54

6. Validation of the moving geometry 61
6.1. Convergence study - Acoustic pulse 62
6.2. Shock capturing - Shock-wall interaction 68
6.2.1. Interaction of a shock wave with a non-moving wall . 68
6.2.2. Interaction of a shock wave with a moving wall . . . 75
6.3. Shock formation - Moving piston 80
6.4. Curved boundary - Moving cylinder 88
6.5. Sharp boundary - Supersonic moving wedge 96
6.6. Reduced computation inside the geometry 101
6.7. Scalability and computational cost of the embedded method 105
6.7.1. Scalability of the embedded method 105
6.7.2. Computational cost of the embedded method 106

7. Multi-scale problems - An efficient strategy 117
7.1. State of the art - Coupled problems 118
7.2. Partitioned coupling 120
7.3. Quality of the solution - Data mapping 121
7.3.1. Interpolation 122
7.3.2. Data mapping by evaluation 125
7.3.3. Error investigation 126

8. Load balancing - Coupled multi-scale problems 141
8.1. Intra-subdomain 142
8.2. SPartA algorithm 145
8.3. Inter-subdomain 147
8.4. Optimization of the gradient computation 148
8.5. Load balancing for a 3-field coupled simulation 151

9. Application examples - Complex moving geometries 157
9.1. Numerical results - Moving geometry 157
9.1.1. Supersonic flow - Moving cylinder 158
9.1.2. Rotating airfoil 162
9.1.3. Collision of two moving spheres 164

9.1.4. Collision of three moving Spheres 168
9.1.5. Rotating fan 171
9.2. Performance results - Moving geometry 174
9.2.1. Rotating fan 174
9.2.2. Collision of three moving Spheres 179

10. Numerical results - Coupled 3-field simulation **181**
10.1. Innermost subdomain: Compressible Navier-Stokes 188
10.2. Innermost and middle subdomain: Compressible Navier-Stokes and Euler 195
10.3. Innermost, middle and outermost subdomain: Navier-Stokes, Euler and Linearized Euler 201
10.4. Coupling interfaces of the 3-field coupled simulation 209
10.5. Performance 213

11. Conclusion **217**

A. Appendix **221**
A.1. Multi-scale simulations - Partitioned coupling with *preCICE* 221
A.2. Reduced computation inside the geometry - Code 223
A.3. Sharp boundary - Supersonic moving wedge 224
A.4. Sponge - Reduced boundary reflections 229

Bibliography **233**

List of Figures **243**

List of Tables **257**