Contents

L.			tical models and the finite element solution. Hier- modeling	1
	1.1		luctory remarks	1
	1.2		ematical models	2
	1.2	1.2.1	A demonstrative problem – a carabiner	3
		1.2.2	-	6
		1.2.3	<u> </u>	U
		1.2.0	biner	7
	1.3	Rema	rks on the hierarchical modeling process	14
	1.4		ne of book	17
	1.7	Outili	ile of book	1.
2.	Fun		ntal steps in structural mechanics	19
	2.1	Gener	ral conditions	19
		2.1.1	Motion of a deformable three-dimensional body	19
		2.1.2	Properly supported bodies	23
		2.1.3	Internal actions	24
		2.1.4	Assumptions for static analysis	25
		2.1.5	Assumptions for a linear static analysis	25
		2.1.6	Summary	26
	2.2	The a	nalysis of truss structures — to exemplify general con-	
		cepts	of analysis	27
		2.2.1	Model assumptions	27
		2.2.2	Kinematic conditions for a properly supported truss	28
		2.2.3	Equilibrium conditions for a truss model	30
		2.2.4	Constitutive behavior for a truss bar	34
		2.2.5	Compatibility conditions for a truss	35
		2.2.6	Statically determinate and indeterminate trusses	40
	2.3	Matri	x displacement method for trusses	44
		2.3.1	Truss bar stiffness matrix in its local system	45
		2.3.2	Solution of a two-bar truss structure using the matrix	
			method	47
		2.3.3	Stiffness matrix of an arbitrarily oriented truss element	52
		2.3.4	Solution of the three-bar truss structure using the ma-	
			trix method	56



		2.3.5	Systematization of the matrix formulation for truss	
			structures	
		2.3.6	Principle of superposition	
		2.3.7	Remarks about the structure stiffness matrix	
		2.3.8	Strain energy of a truss structure	73
		2.3.9	Properly supported truss structures in the context of	
	2.4	Mode	the matrix method	
3.	The	e linea	r 3-D elasticity mathematical model	83
	3.1		analysis of a steel sheet problem	
		3.1.1	One-dimensional conditions	
		3.1.2	Two Dimensional Conditions	
	3.2	Defor	mations	95
		3.2.1	Displacement field	
		3.2.2	Normal and shear strains	
		3.2.3	Finite and infinitesimal rigid deformations	113
		3.2.4	Technical or engineering notation for the strains	119
		3.2.5	Deformation in the vicinity of a point	
	3.3	Stress	ses	
		3.3.1	Classical concept of stress	126
		3.3.2	Characterization of the state of stress at a point	128
		3.3.3	Differential equilibrium equations	136
		3.3.4	Principal stresses	139
		3.3.5	Principal strains	148
		3.3.6	Infinitesimally small displacements	149
		3.3.7	Technical or engineering notation for the stresses	150
	3.4	Const	citutive equations	150
		3.4.1	Hooke's law for three-dimensional isotropic material	
			conditions	151
		3.4.2	Relation between G and E , ν	155
		3.4.3	Generalized Hooke's law for an isotropic material in	
			matrix notation	
	3.5	Form	ulation of the linear elasticity problem	159
	3.6		on of a prismatic bar	
	ъ. г	41	45. 1 11 11 1 1 1 1 1 1 1	
4.			tical models used in engineering structural analy-	
	4.1		·····	
	4.1		elasticity	
		4.1.1	The plane stress model	
		4.1.2	The plane stress model	
	4.0	4.1.3	The axisymmetric model	
	4.2		nodels	
		4.2.1	EDSIDALIC DAT SUDJECTED TO AXIAL IOADING	711

		4.2.2	Prismatic bar subjected to transverse loading; the Bernoulli-Euler beam model
		4.2.3	Bar models obtained by an assemblage of bars 239
		4.2.4	Matrix displacement method for frames 245
		4.2.5	Bars subjected to 3-D actions
		4.2.6	Thin walled bars
		4.2.7	Curved bar model
		4.2.8	The Timoshenko beam model
	4.3		in bending
	4.0	4.3.1	The Kirchhoff plate bending model
		4.3.2	The Reissner-Mindlin plate bending model
	4.4		
	4.4	4.4.1	
		4.4.1	Geometrical preliminaries
			Shell mathematical models
		4.4.3	Shells of revolution loaded axisymmetrically 335
	4 5	4.4.4	Remarks on shell modeling of engineering structures 353
	4.5	Summ	ary of the mathematical models for structural mechanics 354
5.		_	iple of virtual work
	5.1	_	rinciple of virtual work for the bar problem
	5.2	The p	rinciple of virtual work in 2-D and 3-D analyses 380
		5.2.1	The principle of virtual work for 3-D elasticity 381
		5.2.2	The principle of virtual work for the plane stress model 385
		5.2.3	The principle of virtual work for the plane strain model 388
		5.2.4	The principle of virtual work for the axisymmetric
			model
		5.2.5	The principle of virtual work for the Bernoulli-Euler
			beam model
	5.3		and potential energy in 3-D 390
		5.3.1	Strain energy
		5.3.2	The total potential energy
6.	The		element process of solution 395
	6.1	Finite	element formulation of 1-D bar problem 395
	6.2	Conve	rgence properties of 1-D finite element solutions 415
		6.2.1	Convergence conditions 420
		6.2.2	Distances and norms of functions 421
		6.2.3	Convergence properties
		6.2.4	Smoothness of the solution
		6.2.5	The h , p and $h-p$ finite element methods
	6.3		cement-based finite element formulation for solids 434
	J.0	6.3.1	Discretization methodology
		6.3.2	Equilibrium properties of the finite element solutions
		J.U.4	for 2-D
		633	Isoparametric finite elements
			ANTIFICIAL CALLACTURE OF THE CONTROL

		6.3.4	Numerical integration
		6.3.5	Displacement-based finite element formulations for 3-
		6.3.6	D solids
		0.0.0	ements
	6.4	Finite	e elements for beams, plates and shells 461
	0.1	6.4.1	Bernoulli-Euler beam finite element
		6.4.2	Matrix and finite element structural analysis for frames 464
		6.4.3	Plate and shell finite elements
		6.4.4	Beam finite elements
	6.5	Effect	ive finite elements
		6.5.1	Convergence of displacement-based finite element for-
			mulations and the effects of locking 476
		6.5.2	Locking for the 2-node Timoshenko beam element 479
		6.5.3	Locking in a general setting
	6.6	Finite	e element modeling tools
		6.6.1	Combining different type of elements in the same model 489
		6.6.2	Constraint equations
		6.6.3	Rigid links
		6.6.4	Spring elements
		6.6.5	Model symmetries
		6.6.6	Substructures
		6.6.7	Connecting different type of elements 499
		6.6.8	Skew systems
	6.7		ng issues and error assessment
		6.7.1	Mesh grading 503
		6.7.2	Element shape
		6.7.3	Error estimation and adaptative procedures 505
	6.8		element model construction 509
	6.9	A fini	te element modeling example
7.	Hie	rarchi	cal modeling examples
	7.1	Built-	in cantilever subjected to a tip load 519
		7.1.1	Bernoulli-Euler beam model
		7.1.2	Timoshenko beam model
		7.1.3	Plane stress solution
	7.2		ine Tool Jig
			Beam Model
		7.2.2	The Shell Model
		7.2.3	Three-Dimensional Elasticity Model
		7.2.4	Qualitative Analysis
		7.2.5	Quantitative Analysis
	7.3		ling of a carabiner
		7.3.1	Straight bar model
		7.3.2	Curved bar model

				Contents	Х
		7.3.3	Three-dimensional elasticity model		. 5
		7.3.4	Qualitative analysis		
		7.3.5	Quantitative analysis		
8.	Мо	deling	g for nonlinear analysis. An aperçu .		. 5
	8.1	Sourc	es of nonlinearity		. 5
	8.2	Incre	mental formulation for nonlinear analysis	3	. ;
	8.3	Deter	mination of ultimate loads leading to str	uctural collaps	e :
	8.4	Mode	eling nonlinear problems		r