

Contents

Introduction	1
1 Mathematical Foundations	7
1.1 Ordinary Differential Equations	7
1.2 Delay Differential Equations	11
1.3 Conservation Laws	13
2 First Order Model	19
2.1 LWR Model	19
2.2 Derivation of the First Order Delay Model	21
2.3 Properties of the model	23
2.3.1 Conservation	24
2.3.2 Positivity	24
2.3.3 Upper Bound	25
2.3.4 Linear Stability of the discretized systems	26
2.3.5 Comparison to solutions in the undelayed case	27
2.4 Numerical Methods	28
2.4.1 Method of Steps Approach	28
2.4.2 Altered Lax-Friedrichs Method	33
3 Second Order Model	39
3.1 Aw-Rascle-Zhang model	39
3.2 Derivation of the Second Order Delay Model	43
3.2.1 Reverse spatial discretization	43
3.2.2 Coarse-graining	45
3.2.3 Taylor expansion	47
3.3 Properties of the models	48
3.3.1 Conservation	48
3.3.2 Positivity	49
3.3.3 Upper Bound	51
3.3.4 Linear Stability of the discretized systems	53
3.3.5 Comparison to solutions in the undelayed case	54
3.4 Numerical Methods	55

3.4.1	Method of Lines	55
3.4.2	Splitting Method	56
3.5	Other methods for hyperbolic equations	57
3.6	Parameter Estimation	57
3.6.1	Parameter Estimation for Microscopic Models	58
3.6.2	Parameter Estimation for Macroscopic Models	62
4	Simulation Results	65
4.1	Comparison Undelayed and Delayed Models	65
4.2	Comparison Microscopic and Macroscopic Models	89
4.3	Fitting to Real Data	96
	Conclusion	99
	Appendix A	101
	Appendix B	103