

Contents

Preface — VII

1	The classical harmonic oscillator — 1
2	Lagrangian and Hamiltonian mechanics — 9
2.1	Introduction — 9
2.2	State space and Lagrangians — 9
2.3	Phase space and Hamiltonians — 46
2.4	Segue — 64
3	The formalism of quantum mechanics: motivation — 67
3.1	Introduction — 67
3.2	States, observables and dynamics — 68
3.3	Mechanics and Hilbert spaces — 74
4	Physical background — 79
4.1	Introduction — 79
4.2	Electromagnetic radiation — 80
4.3	Blackbody radiation and the photoelectric effect — 96
4.4	Two-slit experiments — 121
5	Synopsis of self-adjoint operators, spectral theory and Stone's theorem — 129
5.1	Introduction — 129
5.2	Unbounded self-adjoint operators — 131
5.3	$H_B = \frac{1}{2m}P^2 + \frac{m\omega^2}{2}Q^2$ — 142
5.4	The spectrum — 155
5.5	The spectral theorem — 165
5.6	Stone's theorem — 180
6	The postulates of quantum mechanics — 189
6.1	Introduction — 189
6.2	The postulates — 189
6.3	Uncertainty relations — 218
6.4	Schrödinger and Heisenberg pictures — 225
7	Canonical quantization — 231
7.1	$PQ - QP = \frac{\hbar}{2\pi i}$ — 231
7.2	Heisenberg algebras and Heisenberg groups — 242
7.3	The free quantum particle — 272

7.4	The quantum harmonic oscillator —	286
7.4.1	Introduction —	286
7.4.2	The classical limit problem —	290
7.4.3	The propagator —	303
8	Path integral quantization —	311
8.1	Motivation —	311
8.2	Path integral for the free quantum particle —	321
8.3	Path integral for the harmonic oscillator —	325
9	Sketches of some rigorous results —	339
9.1	Introduction —	339
9.2	Self-adjointness of $-\frac{\hbar^2}{2m}\Delta + V$ —	343
9.3	Brownian motion and Wiener measure —	358
9.4	Analytic continuation —	383
10	Fermionic and supersymmetric harmonic oscillators —	391
10.1	The Stern–Gerlach experiment and spin one-half —	391
10.2	Anticommutation relations and the fermionic harmonic oscillator —	407
10.3	$N = 2$ supersymmetry and the harmonic oscillator —	425
10.4	$N = 2$ supersymmetry and Hodge theory —	440
A	Gaussian integrals —	455
B	Morse lemma —	465
C	Stationary phase approximation —	473
D	Tangent and cotangent bundles —	481
E	Poisson and wave equations —	483
F	Carathéodory procedure —	485
G	Schwartz space, Fourier transform, distributions and Sobolev spaces —	487
G.1	Introduction —	487
G.2	Dimension 1 —	487
G.3	Heat kernel —	497
G.4	Dimension n —	502

H	Stieltjes integrals — 511
H.1	Introduction — 511
H.2	Riemann–Stieltjes integrals — 511
H.3	Lebesgue–Stieltjes integrals — 512
I	Unitary representations and Schur’s lemma — 517
J	Semigroups of operators — 521
K	Hilbert space tensor products — 529
	Bibliography — 531
	Index — 543