

# Contents

<b>1</b>	<b>Introduction to Grey Systems Theory.....</b>	<b>1</b>
1.1	Appearance and Growth of Grey Systems Research .....	1
1.1.1	The Scientific Background.....	1
1.1.2	The Development History and Current State .....	2
1.1.3	Characteristics of Unascertained Systems.....	5
1.1.3.1	Incomplete Information.....	5
1.1.3.2	Inaccuracies in Data .....	6
1.1.3.3	The Scientific Principle of Simplicity .....	7
1.1.3.4	Precise Models Suffer from Inaccuracies.....	8
1.1.4	Comparison of Several Studies of Uncertain Systems .....	10
1.1.5	Emerging Studies on Uncertain Systems .....	11
1.1.6	Position of Grey Systems Theory in Cross- Disciplinary Researches .....	13
1.2	Basics of Grey Systems .....	15
1.2.1	Elementary Concepts of Grey Systems .....	15
1.2.2	Fundamental Principles of Grey Systems .....	16
1.2.3	Main Components of Grey Systems Theory .....	16
<b>2</b>	<b>Basic Building Blocks.....</b>	<b>19</b>
2.1	Grey Numbers, Degree of Greyness, and Whitenization.....	19
2.1.1	Grey Numbers .....	19
2.1.2	Whitenization of Grey Numbers and Degree of Greyness .....	24
2.1.3	Degree of Greyness Defined by Using Axioms .....	27
2.2	Sequence Operators .....	29
2.2.1	Systems under Shocking Disturbances and Sequence Operators.....	30
2.2.2	Axioms That Define Buffer Operators .....	31
2.2.3	Properties of Buffer Operators .....	32
2.2.4	Construction of Practically Useful Buffer Operators .....	33
2.3	Generation of Grey Sequences .....	42
2.3.1	Average Generator .....	42

- 2.3.2 Smoothness of Sequences .....43
    - 2.3.3 Stepwise Ratio Generator.....45
    - 2.3.4 Accumulating and Inverse Accumulating Generators.....46
  - 2.4 Exponentiality of Accumulating Generations.....48
- 3 Grey Incidence and Evaluations..... 51**
  - 3.1 Grey Incidence and Degree of Grey Incidences .....52
    - 3.1.1 Grey Incidence Factors and Set of Grey Incidence Operators .....52
    - 3.1.2 Metric Spaces.....54
    - 3.1.3 Degrees of Grey Incidences .....55
  - 3.2 General Grey Incidences.....57
    - 3.2.1 Absolute Degree of Grey Incidence .....57
    - 3.2.2 Relative Degree of Grey Incidence .....61
    - 3.2.3 Synthetic Degree of Grey Incidence .....62
  - 3.3 Grey Incidence Models Based on Similarity and Closeness.....63
  - 3.4 Grey Cluster Evaluations .....68
    - 3.4.1 Grey Incidence Clustering.....68
    - 3.4.2 Grey Variable Weight Clustering.....70
    - 3.4.3 Grey Fixed Weight Clustering .....72
  - 3.5 Grey Evaluation Using Triangular Whitenization Functions.....75
    - 3.5.1 Evaluation Model Using Endpoint Triangular Whitenization Functions .....75
    - 3.5.2 Evaluation Model Using Center-Point Triangular Whitenization Functions .....81
    - 3.5.3 Comparison between Evaluation Models of Triangular Whitenization Functions .....83
  - 3.6 Applications.....90
    - 3.6.1 Order of Grey Incidences .....90
    - 3.6.2 Preference Analysis.....91
    - 3.6.3 Practical Applications.....92
- 4 Grey Systems Modeling..... 107**
  - 4.1 The GM(1,1) Model.....107
    - 4.1.1 The Basic Form of GM(1,1) Model .....107
    - 4.1.2 Expanded Forms of GM(1,1) Model.....109
  - 4.2 Improvements on GM(1,1) Models .....116
    - 4.2.1 Remnant GM(1,1) Model.....116
    - 4.2.2 Groups of GM(1,1) Models.....118
  - 4.3 Applicable Ranges of GM(1,1) Models.....119
  - 4.4 The GM(r,h) Models.....123
    - 4.4.1 The GM(1, N) Model .....123
    - 4.4.2 The GM(0,N) Model .....125
    - 4.4.3 The GM(2,1) and Verhulst Models .....125

4.4.3.1	The GM(2,1) Model.....	125
4.4.3.2	The Verhulst Model.....	127
4.4.4	The GM(r,h) Models .....	130
4.5	Grey Systems Predictions .....	133
4.5.1	Sequence Predictions .....	135
4.5.2	Interval Predictions .....	135
4.5.3	Disaster Predictions.....	139
4.5.3.1	Grey Disaster Predictions .....	139
4.5.3.2	Seasonal Disaster Predictions .....	140
4.5.4	Stock-Market-Like Predictions .....	141
4.5.5	Systems Predictions .....	145
4.5.5.1	The Thought of Five-Step Modeling .....	145
4.5.5.2	System of Prediction Models.....	147
<b>5</b>	<b>Discrete Grey Prediction Models.....</b>	<b>149</b>
5.1	The Basics.....	149
5.1.1	Definitions on Discrete Grey Models .....	149
5.1.2	Relationship between Discrete Grey and GM(1,1) Models .....	152
5.1.3	Prediction Analysis of Completely Exponential Growths .....	154
5.2	Generalization and Optimization of Discrete Grey Models.....	156
5.2.1	Three Forms of Discrete Grey Models .....	156
5.2.2	Impacts of Initial Values on Iterations.....	157
5.2.3	Optimization of Discrete Grey Models.....	159
5.2.4	Recurrence Functions for Optimizing Discrete Grey Models.....	161
5.3	Approximately Nonhomogeneous Exponential Growth.....	163
5.4	Discrete Grey Models of Multi-variables .....	166
<b>6</b>	<b>Combined Grey Models.....</b>	<b>169</b>
6.1	Grey Econometrics Models .....	169
6.1.1	Determination of Variables Using Principles of Grey Incidence .....	169
6.1.2	Grey Econometrics Models.....	170
6.2	Combined Grey Linear Regression Models.....	173
6.3	Grey Cobb-Douglas Model.....	177
6.4	Grey Artificial Neural Network Models .....	178
6.4.1	BP Artificial Neural Model and Computational Schemes.....	178
6.4.2	Principle and Method for Grey BP Neural Network Modeling .....	179
6.5	Grey Markov Model .....	181
6.5.1	Grey Moving Probability Markov Model .....	181
6.5.2	Grey State Markov Model.....	183
6.6	Combined Grey-Rough Models.....	183
6.6.1	Rough Membership, Grey Membership and Grey Numbers.....	184
6.6.2	Grey Rough Approximation.....	187
6.6.3	Combined Grey Clustering and Rough Set Model.....	190

<b>7</b>	<b>Grey Models for Decision Making.....</b>	<b>197</b>
7.1	Different Approaches for Grey Decisions .....	198
7.1.1	Grey Target Decisions.....	198
7.1.2	Grey Incidence Decisions.....	203
7.1.3	Grey Development Decisions.....	207
7.1.4	Grey Cluster Decisions .....	210
7.2	Decision Makings with Synthesized Targets.....	214
7.3	Multi-attribute Intelligent Grey Target Decision Models .....	218
<b>8</b>	<b>Grey Game Models.....</b>	<b>225</b>
8.1	Strategic Game Models for Duopolies with Limited Rationality and Knowledge .....	226
8.1.1	Duopolistic Strategic Output-Making Models Based on Empirically Ideal Production and Optimal Decision Coefficients .....	226
8.1.2	Concession Equilibrium of the Later Decision-Maker under Nonstrategic Expansion Damping Conditions: Elimination from the Market .....	230
8.1.3	Damping Equilibrium of the Advanced Decision-Maker under Strategic Expansion Damping Conditions: Giving Up Some Market Share.....	233
8.1.4	Damping Loss and Total Damping Cost for the First Decision-Making Oligopoly to Completely Control the Market .....	237
8.2	A New Situational Forward Induction Model.....	244
8.2.1	Weaknesses of Backward Induction, Central Method of Equilibrium Analysis for Dynamic Games .....	244
8.2.2	Backward Derivation of Multi-Stage Dynamic Games' Profits.....	245
8.2.3	Termination of Forward Induction of Multi-Stage Dynamic Games and Guide Nash Equilibrium Analysis .....	248
8.3	Chain Structure Model of Evolutionary Games of Industrial Agglomerations and Its Stability .....	252
8.3.1	Chained Evolutionary Game Model for the Development of Industrial Agglomerations .....	252
8.3.2	Duplicated Dynamic Simulation for the Development Process of Industrial Agglomerations .....	255
8.3.3	Stability Analysis for the Formation and Development of Industrial Agglomerations .....	257
<b>9</b>	<b>Grey Control Systems .....</b>	<b>259</b>
9.1	Controllability and Observability of Grey Systems.....	260
9.2	Transfer Functions of Grey Systems.....	262
9.2.1	Grey Transfer Functions .....	262

9.2.2	Transfer Functions of Typical Links .....	263
9.2.3	Matrices of Grey Transfer Functions .....	266
9.3	Robust Stability of Grey Systems .....	268
9.3.1	Robust Stability of Grey Linear Systems .....	268
9.3.2	Robust Stability of Grey Linear Time-Delay Systems .....	271
9.3.3	Robust Stability of Grey Stochastic Linear Time-Delay Systems .....	273
9.4	Several Typical Grey Controls .....	278
9.4.1	Control with Abandonment .....	278
9.4.2	Control of Grey Incidences .....	279
9.4.3	Control of Grey Predictions .....	280
<b>10</b>	<b>Introduction to Grey Systems Modeling Software.....</b>	<b>287</b>
10.1	Features and Functions .....	288
10.2	Main Components.....	290
10.3	Operation Guide.....	291
10.3.1	The Confirmation System .....	291
10.3.2	Using the Software Package.....	293
10.3.2.1	Entering Data.....	294
10.3.2.2	Model Computations.....	295
<b>A</b>	<b>Interval Analysis and Grey Systems Theory .....</b>	<b>303</b>
A.1	A Brief Historical Account of Interval Analysis.....	303
A.2	Main Blocks of Interval Analysis .....	304
A.2.1	Interval Number System and Arithmetic.....	305
A.2.2	Interval Functions, Sequences and Matrices .....	306
A.2.3	Interval Newton Methods.....	308
A.2.4	Integration of Interval Functions.....	309
	References .....	313
<b>B</b>	<b>Approaches of Uncertainty .....</b>	<b>315</b>
B.1	Foundation for a Unified Information Theory .....	316
B.1.1	Grey Uncertainties.....	317
B.1.2	Stochastic Uncertainties .....	317
B.1.3	Unascertainties .....	317
B.1.4	Fuzzy Uncertainties.....	317
B.1.5	Rough Uncertainties.....	318
B.1.6	Soros Reflexive Uncertainties .....	318
B.2	Relevant Practical Uncertainties .....	318
B.3	Some Final Words and Open Questions .....	323
	References .....	324

<b>C</b>	<b>How Uncertainties Appear: A General Systems Approach.....</b>	<b>325</b>
C.1	Evolutionary Transitions.....	325
C.1.1	Blown-Ups: Old Structures Replaced by New Ones .....	325
C.1.2	Mathematical Properties of Blown-Ups .....	327
C.1.3	The Problem of Quantitative Infinity.....	328
C.1.4	Eddy Motions of the General Dynamic System .....	329
C.1.5	Equal Quantitative Effects.....	330
C.2	The Systemic Yoyo Structure of General Systems .....	333
C.2.1	The Systemic Yoyo Model.....	333
C.2.2	Justification Using Conservation Law of Informational Infrastructures .....	334
C.2.3	Justification Using Readily Repeatable Experiments .....	334
C.3	Laws on State of Motion of Systems .....	335
C.3.1	The Quark Structure of Systemic Yoyos .....	336
C.3.2	Interactions between Systemic Yoyos.....	338
C.3.3	Laws on State of Motion .....	340
C.4	Uncertainties Everywhere .....	343
C.4.1	Artificial and Physical Uncertainties .....	343
C.4.2	Uncertainties That Exist in the System of Modern Mathematics .....	344
C.4.2.1	Uncertainties of Mathematics.....	344
C.4.2.2	Inconsistencies in the System of Mathematics .....	345
C.5	A Few Final Words.....	348
References	.....	348
<b>References.....</b>		<b>351</b>
<b>Index.....</b>		<b>373</b>