

# Contents

<b>New Applications of Mortar Methodology to Extended and Embedded Finite Element Formulations</b> .....	1
<i>Tod A. Laursen, Jessica D. Sanders</i>	
1 Introduction .....	1
2 Stability Issues Associated with Contact on Enriched Interfaces .....	2
3 Adaptation to the Embedded Interface Case .....	4
4 Conclusion .....	7
References .....	8
<b>Thermo-Mechanical Coupling in Beam-to-Beam Contact</b> ....	9
<i>Daniela P. Boso, Przemyslaw Litewka, Bernhard A. Schrefler</i>	
1 Thermo-Mechanical Beam Finite Element .....	9
2 Weak Form for Thermo-Mechanical Contact .....	11
3 Numerical Example .....	13
4 Conclusions .....	15
References .....	15
<b>On Regularization of the Convergence Path for the Implicit Solution of Contact Problems</b> .....	17
<i>Giorgio Zavarise, Laura De Lorenzis, Robert L. Taylor</i>	
1 Introduction .....	17
2 Structure of the Consistent Tangent Stiffness .....	19
3 Large Penetration Basic Algorithm .....	20
3.1 Strategy Outline .....	20
3.2 Modified Stiffness and Residual during Phase One ...	21
3.3 Limitations of the Strategy .....	22
4 Large Penetration Enhanced Algorithm .....	23
4.1 Solution of the Problem for $r < 1$ .....	23

5	Example .....	25
6	Conclusions.....	27
	References .....	28
<b>On Different Variational Formulations of the Nitsche</b>		
	<b>Method</b> .....	29
<i>Ridvan Izi, Alexander Konyukhov, Karl Schweizerhof</i>		
1	Nitsche Formulation .....	29
1.1	Choice of the Lagrange Multiplier Set $\mu$ .....	31
1.2	Physical Meaning of the Non-penetration Terms.....	32
2	Types of the Nitsche Approach .....	33
3	FE Implementation of the Nitsche Approaches .....	34
3.1	Gauss Point-Wise Substituted Formulation .....	34
3.2	Bubnov-Galerkin-Wise Partial Substituted Formulation .....	35
4	Numerical Example .....	36
	References .....	38
<b>Challenges in Computational Nanoscale Contact</b>		
	<b>Mechanics</b> .....	39
<i>Roger A. Sauer</i>		
1	Introduction .....	39
2	Nanoscale Contact Challenges.....	39
3	Nanoscale versus Macroscale Contact .....	40
4	Adhesion Instability .....	42
5	Multiscale Contact Modeling.....	44
6	Conclusion .....	45
	References .....	45
<b>On the Four-node Quadrilateral Element</b> .....		
	<i>Ulrich Hueck, Peter Wriggers</i>	47
1	Introduction .....	47
2	Element Formulation .....	48
3	Numerical Example.....	49
	References .....	49
<b>Stability of Mixed Finite Element Formulations – A New</b>		
	<b>Approach</b> .....	51
<i>Stefanie Reese, Vivian Tini, Yalin Kiliçlar, Jan Frischkorn,</i>		
<i>Marco Schwarze</i>		
1	Introduction .....	51
2	Linear Elasticity - Mixed Variational Formulation .....	52
3	Interpolation.....	53
3.1	Compatible Strain.....	53
3.2	Enhanced Strain .....	54
4	Element Stiffness Matrix .....	54

5	Eigenvalue Analysis .....	55
6	Non-linear Finite Element Technology .....	55
	References .....	59
<b>A Finite Element Formulation Based on the Theory of a Cosserat Point – Modification of the Torsional Modes .....</b>		<b>61</b>
<i>Eiris F.I. Boerner, Dana Mueller-Hoeppe, Stefan Loehnert</i>		
1	Motivation .....	61
2	A Brief Introduction to the Cosserat Point Element .....	62
	2.1 Kinematics .....	62
	2.2 Equilibrium .....	63
	2.3 Constitutive Equations .....	64
3	Torsion .....	65
4	Conclusions .....	68
	References .....	68
<b>A Brick Element for Finite Deformations with Inhomogeneous Mode Enhancement .....</b>		<b>69</b>
<i>Dana Mueller-Hoeppe, Stefan Loehnert</i>		
1	Introduction .....	69
2	Theoretical Background .....	70
	2.1 Enhanced Strain Assumption .....	71
	2.2 Variational Formulation .....	72
3	Discretization .....	72
4	Numerical Examples .....	73
	4.1 Irregularly Meshed Beam .....	73
	4.2 Nearly Incompressible Block .....	74
5	Conclusions .....	76
	References .....	76
<b>Automatic Differentiation Based Formulation of Computational Models .....</b>		<b>79</b>
<i>Jože Korelc</i>		
1	Introduction .....	79
2	Automatic Differentiation .....	80
3	Automatic Differentiation in Computational Mechanics .....	81
4	Automatic Differentiation Based Computational Models .....	82
	4.1 ADB Form of Hyperelastic Models .....	82
	4.2 ADB Form of Elasto-plastic Models .....	82
	4.3 Numerical Efficiency of ADB Form .....	83
	4.4 ADB Form of Contact Formulations .....	84
	4.5 ADB Form in Stability Analysis .....	85
5	Conclusions .....	85
	References .....	86

<b>Nonlinear Finite Element Shell Formulation Accounting for Large Strain Material Models</b> .....	87
<i>Friedrich Gruttmann</i>	
1 Introduction .....	87
2 Variational Formulation of the Shell Equations .....	88
3 Mixed Hybrid Shell Element .....	90
4 Numerical Example: Stretching of a Rubber Sheet .....	91
5 Conclusions .....	94
References .....	94
<b>Hybrid and Mixed Variational Principles for the Geometrically Exact Analysis of Shells</b> .....	97
<i>Paulo de Mattos Pimenta</i>	
1 Introduction .....	97
2 The Geometrically-Exact First-Order-Shear Shell Model ....	98
3 Some Multi-field Variational Principles .....	103
3.1 Principle of Total Potential Energy .....	103
3.2 Three-Field Principle of Veubeke-Hu-Washizu Type .....	103
3.3 Two-Field Principle of Hellinger-Reissner Type .....	104
3.4 Two-Field Principle of Total Complementary Potential Energy .....	104
3.5 Hybrid Principle of Hellinger-Reissner Type .....	105
References .....	106
<b>A Shell Theory with Scale Effects, Higher Order Gradients, and Meshfree Computations</b> .....	107
<i>Carlo Sansour, Sebastian Skatulla</i>	
1 Introduction .....	107
2 Deformation and Strain .....	108
3 Generalized Shell Theory .....	109
4 Numerical Example .....	111
References .....	114
<b>An Electro-mechanically Coupled FE-Formulation for Piezoelectric Shells</b> .....	115
<i>W. Wagner, K. Schulz, and S. Klinkel</i>	
1 Introduction .....	116
2 Kinematics .....	116
3 Constitutive Equations .....	117
4 Finite Element Approximation .....	118
5 Numerical Example .....	120
References .....	123

<b>Non-intrusive Coupling: An Attempt to Merge Industrial and Research Software Capabilities</b> .....	125
<i>Olivier Allix, Lionel Gendre, Pierre Gosselet, Guillaume Guguin</i>	
1 Introduction .....	126
2 The General Principles of Non-intrusive Coupling.....	126
2.1 Piecewise Substitution .....	127
2.2 Iterative Coupling .....	128
2.3 Choice of the Interface Boundary Condition for the Local Step.....	130
3 Examples Using Abaqus/Standard .....	130
4 Conclusion .....	132
References .....	132
<b>Constitutive Models and Failure Prediction for Al-Alloys in Industrial Applications</b> .....	135
<i>Christian Leppin</i>	
1 Introduction .....	135
2 Factors Influencing Properties .....	136
3 Work-Hardening of Aluminum Alloys .....	136
4 Yield Locus.....	138
5 Fracture Prediction .....	140
6 Conclusions.....	141
References .....	142
<b>A Phenomenological Damage Model to Predict Material Failure in Crashworthiness Applications</b> .....	143
<i>Markus Feucht, Frieder Neukamm, and André Haufe</i>	
1 Introduction .....	144
2 The Process Chain of Sheet Metal Part Manufacturing .....	144
3 Failure Modelling in Forming and Crashworthiness Simulations .....	144
3.1 A Generalized Scalar Damage Model.....	146
3.2 Failure Prediction .....	147
4 Path-Dependent Localization .....	147
4.1 Stress and Strain Measures .....	148
4.2 Nonlinear Accumulation of the Instability Criterion .....	149
5 Post Critical Behaviour .....	150
5.1 Damage-Dependent Yield Stress.....	151
5.2 Energy Dissipation and Fadeout.....	151
6 Conclusions.....	152
References .....	153

<b>A Computational Approach for Mixed-Lubrication Effects in Sealing Applications</b> .....	155
<i>Markus André</i>	
1 Introduction .....	155
2 Basic Equations .....	156
2.1 Solid Mechanics .....	156
2.2 Fluid Mechanics .....	157
3 Coupled Fluid Film Computation .....	159
4 Friction Approach .....	160
5 Example .....	162
References .....	162
<b>Deformations of a Large Hall: Structural Design and Analysis</b> .....	163
<i>Klaus-Dieter Klee, Reinhard Kahn</i>	
1 Introduction .....	163
2 Steel Construction .....	164
2.1 Bearing Structure .....	165
2.2 Roof .....	166
2.3 Stiffening Components .....	168
2.4 Support of Partial Halls .....	168
3 Construction and Computation .....	171
4 Summary .....	177
References .....	177
<b>Recovering Micropolar Continua from Particle Mechanics by Use of Homogenisation Strategies</b> .....	179
<i>Wolfgang Ehlers</i>	
1 Introduction .....	179
2 The Particle Model .....	180
3 Homogenisation Technique .....	183
4 Numerical Example .....	186
5 Conclusion .....	188
References .....	189
<b>Modelling of Microstructured Materials with Micromorphic Continuum Approaches</b> .....	191
<i>C. Britta Hirschberger, Paul Steinmann</i>	
1 Introduction .....	191
2 The Micromorphic Continuum .....	192
2.1 Micromorphic Continuum Framework .....	192
2.2 Hyperelastic Constitutive Framework .....	193
2.3 Numerical Aspects .....	193
3 Application to Material Interfaces with Heterogeneous Micromorphic Mesostructure .....	195

3.1	Scale Transition between Interface and Micromorphic RVE .....	195
3.2	A Computational Homogenization Approach for Micromorphic Meso-heterogeneous Material Layers .....	196
3.3	Numerical Examples .....	196
4	Conclusion .....	197
	References .....	198

**On Computational Homogenisation of Heterogeneous Media with Debonded Inclusions .....**

*D. Perić, D.D. Somer, E.A. de Souza Neto, W. Dettmer*

1	Introduction and Background .....	199
2	Multi-scale Constitutive Theory: Overview .....	200
2.1	RVE Kinematical Constraints .....	201
2.2	Finite Element Approximation .....	201
2.3	Solution Procedure .....	202
3	Frictional Contact .....	202
3.1	Boundary Value Problem .....	202
3.2	Constitutive Relations .....	202
4	Assessment of Yield Surfaces of Heterogeneous Media with Debonded Inclusions .....	203
4.1	Computational Homogenisation Based Methodology .....	204
4.2	Estimated Yield Surfaces .....	205
5	Conclusion and Remarks .....	206
	References .....	206

**Assessment of Homogenization Errors in Transient Problems .....**

*K. Runesson, F. Su, F. Larsson*

1	Introduction .....	207
2	Transient Heat Flow – A Model Problem .....	208
2.1	Space-Variational Format .....	208
2.2	Explicit Homogenization Results .....	209
3	RVE-Problem .....	210
3.1	Dirichlet Boundary Conditions .....	210
3.2	Neumann Boundary Conditions .....	211
4	Computational Results .....	212
4.1	Problem Definition – Substructure Characteristics .....	212
5	Conclusions .....	214
	References .....	214

<b>Multiscale Modeling of Metal Foams Using the XFEM</b> . . . . .	215
<i>Lovre Krstulovic-Opara, Stefan Loehnert, Dana Mueller-Hoeppe, Matej Vesenjak</i>	
1 Introduction . . . . .	216
2 Modified XFEM for Heterogeneous Materials . . . . .	216
3 Incorporation of Finite Plasticity . . . . .	218
4 Comparison of Metal Foams with and without Filler Material . . . . .	218
5 Conclusions . . . . .	221
References . . . . .	221
<b>3D Multiscale Projection Method for Micro-/Macrocrack Interaction Simulations</b> . . . . .	223
<i>Stefan Loehnert, Dana Mueller-Hoeppe</i>	
1 Introduction . . . . .	223
2 The Multiscale Technique in Three Dimensions . . . . .	224
2.1 Stress Projection from the Fine Scale to the Coarse Scale . . . . .	224
2.2 Projection of the Displacement Field from the Coarse Scale to the Fine Scale . . . . .	227
3 Numerical Investigations . . . . .	228
4 Conclusion and Outlook . . . . .	230
References . . . . .	230
<b>Goal-Oriented Residual Error Estimates for XFEM Approximations in LEFM</b> . . . . .	231
<i>Marcus Rüter, Erwin Stein</i>	
1 Introduction . . . . .	231
2 XFEM Approximations in LEFM . . . . .	232
2.1 The Model Problem of LEFM . . . . .	232
2.2 XFEM Approximations . . . . .	233
3 A Posteriori Error Estimation in the Energy Norm . . . . .	234
3.1 Error Representation . . . . .	234
3.2 An Implicit Residual Error Estimator . . . . .	234
3.3 Equilibration of Tractions . . . . .	235
4 Goal-Oriented Error Estimation in LEFM . . . . .	236
4.1 Linearization of the $J$ -Integral . . . . .	236
4.2 Duality Techniques . . . . .	236
5 Numerical Example . . . . .	237
6 Conclusions . . . . .	238
References . . . . .	238

<b>Multi-field Coupling Strategies for Large Scale Particle-Fluid Problems</b> .....	239
<i>D.R.J. Owen, Y.T. Feng, K. Han, C.R. Leonardi</i>	
1 Introduction .....	240
2 LB Formulations for Turbulent Incompressible Fluid Flows .....	241
2.1 Standard LB Formulation .....	241
2.2 Turbulence Modelling .....	242
2.3 Hydrodynamic Forces for Fluid-Particle Interactions .....	243
2.4 Fine Particle Modelling - Non-newtonian Fluid Flow .....	243
3 The Thermal Lattice Boltzmann Method .....	244
4 Numerical Illustrations .....	245
4.1 Particle Transportation in Turbulent Fluid Flows .....	245
4.2 Fine Particle Migration in a Block Cave .....	245
4.3 Modelling Heat Transfer in (Particle-)Fluid Flows .....	247
5 Conclusions .....	248
References .....	248
 <b>Numerical Simulation of Particle-Fluid Systems</b> .....	 249
<i>Bircan Avci, Peter Wriggers</i>	
1 Introduction .....	249
2 Mathematical Description .....	250
2.1 Equations for Fluid Motion .....	250
2.2 Equations for Particle Motion .....	250
3 The Discrete Element Model .....	251
3.1 Collision Model for Normal Contact .....	251
3.2 Frictional Tangential Contact Model .....	252
4 Coupling of the Fluid and Particle Phase .....	253
4.1 Evaluation of the Hydrodynamic Forces .....	253
4.2 Coupling Constraints .....	254
5 Numerical Example .....	255
6 Conclusion .....	255
References .....	255
 <b>A Concurrent Multiscale Approach to Non-cohesive Granular Materials</b> .....	 257
<i>Christian Wellmann, Peter Wriggers</i>	
1 Introduction .....	257
2 Discrete Element Method .....	258
3 Homogenization and Elasto-plastic Parameters .....	259
4 Coupling .....	261

5	Numerical Examples .....	262
6	Conclusion .....	264
	References .....	264
<b>On Some Features of a Polygonal Discrete Element Model...</b>		265
<i>Ekkehard Ramm, Manfred Bischoff, Benjamin Schneider</i>		
1	Introduction .....	265
2	Discrete Element Method with Polygonal Particles .....	266
	2.1 Models for Contact .....	266
	2.2 Models for Cohesion .....	268
3	Examples .....	269
	3.1 Model Material without Cohesion .....	269
	3.2 Model Material with Cohesion .....	270
	3.3 Concrete with Microstructure .....	271
4	Conclusions .....	272
	References .....	272
<b>Isogeometric Failure Analysis .....</b>		275
<i>Clemens V. Verhoosel, Michael A. Scott, Michael J. Borden, René de Borst, Thomas J.R. Hughes</i>		
1	Introduction .....	275
2	Isogeometric Finite Elements .....	276
3	Higher-Order Gradient Damage Formulation .....	277
	3.1 Constitutive Behavior .....	278
	3.2 L-Shaped Specimen .....	278
4	Cohesive Zone Formulation .....	279
	4.1 Constitutive Behavior .....	280
	4.2 Single-Edge Notched Beam .....	280
5	Conclusions .....	281
	References .....	282
<b>A Method for Enforcement of Dirichlet Boundary Conditions in Isogeometric Analysis .....</b>		283
<i>Toby J. Mitchell, Sanjay Govindjee, Robert L. Taylor</i>		
1	Introduction .....	284
2	Dirichlet Boundary Conditions .....	285
3	Examples from Linear Elasticity .....	287
	3.1 Infinite Half-Space .....	288
	3.2 Infinite Plate with Circular Hole under Tension .....	289
	3.3 Infinite Plate with Elliptical Hole under Tension .....	291
4	Closure .....	292
	References .....	293

<b>Application of Isogeometric Analysis to Computational Contact Mechanics</b> .....	295
<i>İlker Temizer</i>	
1 Introduction .....	295
2 Contact Boundary Value Problem .....	296
3 Isogeometric Treatment with NURBS .....	296
4 Knot-to-Surface Contact Algorithm .....	298
4.1 Contact of a Grosch Wheel .....	299
4.2 Contact of Two Deformable Bodies .....	300
5 Conclusion .....	300
References .....	302
<b>Stochastic Galerkin Method for the Elastoplasticity Problem with Uncertain Parameters</b> .....	303
<i>Bojana V. Rosic, Hermann G. Matthies</i>	
1 Introduction .....	303
2 Mathematical Formulation .....	304
2.1 Problem Setting .....	304
2.2 Variational Formulation .....	305
3 Numerical Analysis of the Problem .....	305
3.1 Discretisation of Input .....	306
3.2 Stochastic Galerkin Method .....	306
4 Numerical Results .....	307
5 Conclusion .....	310
References .....	310
<b>A Time-Discontinuous Galerkin Approach for the Numerical Solution of the Fokker-Planck Equation</b> .....	311
<i>Udo Nackenhorst, Friederike Loerke</i>	
1 Introduction .....	312
2 FPE Expression of Stochastic Dynamic Problems .....	313
3 Numerical Solution of the Fokker-Planck Equation with TDG Methods .....	314
4 Numerical Example .....	317
5 Conclusions .....	317
References .....	318
<b>Interface Modelling in Computational Limit Analysis</b> .....	321
<i>A.V. Lyamin, K. Krabbenhøft, S.W. Sloan</i>	
1 Discrete Formulation of Bound Theorems .....	321
2 Velocity Discontinuities as a Patch of Thin Elements .....	323
3 Stress Discontinuities as a Patch of Thin Elements .....	324
4 Interfaces between Material Domains .....	325
5 Interfaces at Segments Subject to Loading or Boundary Conditions .....	326

6	Moment-Free Interfaces for Modelling of Joints . . . . .	328
7	Interfaces for Overlapping Connections . . . . .	329
8	Conclusions . . . . .	330
	References . . . . .	330
	<b>On the Coexistence of Intermeshed Hostile Populations . . . . .</b>	<b>331</b>
	<i>Tarek I. Zohdi</i>	
1	Introduction . . . . .	332
	1.1 Objectives . . . . .	333
2	Direct Interaction Models: Rules of Engagement . . . . .	333
3	An Example . . . . .	334
4	Identification of System Parameters: Genetic Algorithms . . .	335
5	An Example of Parity Identification . . . . .	338
6	Concluding Remarks . . . . .	339
	References . . . . .	340