

Contents

Preface	vii
I Convex Analysis	1
1 Linear Spaces	3
1.1 Linear Spaces and Convex Sets	3
1.2 Partially Ordered Linear Spaces	12
1.3 Topological Linear Spaces	21
1.4 Some Examples	32
Notes	35
2 Maps on Linear Spaces	37
2.1 Convex Maps	37
2.2 Differentiable Maps	45
Notes	59
3 Some Fundamental Theorems	61
3.1 Zorn's Lemma and the Hahn-Banach Theorem	61
3.2 Separation Theorems	71
3.3 A James Theorem	81
3.4 Two Krein-Rutman Theorems	87
3.5 Contingent Cones and a Lyusternik Theorem	90
Notes	99

II	Theory of Vector Optimization	101
4	Optimality Notions	103
	Notes	113
5	Scalarization	115
	5.1 Necessary Conditions for Optimal Elements of a Set . .	115
	5.2 Sufficient Conditions for Optimal Elements of a Set . .	129
	5.3 Parametric Approximation Problems	139
	Notes	147
6	Existence Theorems	149
	Notes	159
7	Generalized Lagrange Multiplier Rule	161
	7.1 Necessary Conditions for Minimal and Weakly Minimal Elements	161
	7.2 Sufficient Conditions for Minimal and Weakly Minimal Elements	174
	7.2.1 Generalized Quasiconvex Maps	174
	7.2.2 Sufficiency of the Generalized Multiplier Rule .	181
	Notes	187
8	Duality	189
	8.1 A General Duality Principle	189
	8.2 Duality Theorems for Abstract Optimization Problems	192
	8.3 Specialization to Abstract Linear Optimization Problems	200
	Notes	207
III	Mathematical Applications	209
9	Vector Approximation	211
	9.1 Introduction	211
	9.2 Simultaneous Approximation	213
	9.3 Generalized Kolmogorov Condition	216
	9.4 Nonlinear Chebyshev Vector Approximation	218
	9.5 Linear Chebyshev Vector Approximation	226

9.5.1	Duality Results	227
9.5.2	An Alternation Theorem	233
Notes		241
10	Cooperative n Player Differential Games	243
10.1	Basic Remarks on the Cooperation Concept	243
10.2	A Maximum Principle	245
10.2.1	Necessary Conditions for Optimal and Weakly Optimal Controls	247
10.2.2	Sufficient Conditions for Optimal and Weakly Optimal Controls	259
10.3	A Special Cooperative n Player Differential Game . . .	270
Notes		277
IV	Engineering Applications	279
11	Theoretical Basics of Multiobjective Optimization	281
11.1	Basic Concepts	281
11.2	Special Scalarization Results	291
11.2.1	Weighted Sum Approach	292
11.2.2	Weighted Chebyshev Norm Approach	304
11.2.3	Special Scalar Problems	307
Notes		311
12	Numerical Methods	315
12.1	Modified Polak Method	315
12.2	Eichfelder-Polak Method	321
12.3	Interactive Methods	325
12.3.1	Modified STEM Method	326
12.3.2	Method of Reference Point Approximation . . .	330
The Linear Case		332
The Bicriterial Nonlinear Case		337
12.4	Method for Discrete Problems	343
Notes		348

13 Multiobjective Design Problems	351
13.1 Design of Antennas	352
13.2 Design of FDDI Computer Networks	359
13.2.1 A Cooperative Game	360
13.2.2 Minimization of Mean Waiting Times	362
13.2.3 Numerical Results	365
13.3 Fluidized Reactor-Heater System	367
13.3.1 Simplification of the Constraints	369
13.3.2 Numerical Results	371
13.4 A Cross-Current Multistage Extraction Process	373
13.5 Field Design of a Magnetic Resonance System	376
Notes	380
 V Extensions to Set Optimization	 383
14 Basic Concepts and Results of Set Optimization	385
Notes	391
 15 Contingent Epiderivatives	 393
15.1 Contingent Derivatives and Contingent Epiderivatives	393
15.2 Properties of Contingent Epiderivatives	397
15.3 Contingent Epiderivatives of Real-Valued Functions	401
15.4 Generalized Contingent Epiderivatives	405
Notes	409
 16 Subdifferential	 411
16.1 Concept of Subdifferential	411
16.2 Properties of the Subdifferential	413
16.3 Weak Subgradients	417
Notes	421
 17 Optimality Conditions	 423
17.1 Optimality Conditions with Contingent Epiderivatives	423
17.2 Optimality Conditions with Subgradients	428
17.3 Optimality Conditions with Weak Subgradients	429
17.4 Generalized Lagrange Multiplier Rule	431
17.4.1 A Necessary Optimality Condition	432

17.4.2 A Sufficient Optimality Condition	441
Notes	446
Bibliography	449
List of Symbols	475
Index	477