

Contents

Acknowledgments	vii
Introduction	1
1 Linear Systems and Polynomial Matrices	7
1.1 Definitions and Basics	7
1.2 Networks of Linear Systems	13
2 Counting Problems	21
2.1 General Counting Strategies	22
2.2 Counting Right Prime Polynomial Matrices	29
2.3 Counting Pairwise Coprime Polynomials	38
2.4 Counting Mutually Coprime Polynomial Matrices	55
2.5 Conclusion	81
3 Probabilities of Reachability and Observability for Interconnected Linear Systems	83
3.1 Probabilities for a Single System	84
3.1.1 Reachability and Observability	84
3.1.2 Minimality	85
3.2 Parallel Connection	86
3.3 Series Connection	91
3.3.1 SISO-Systems	92
3.3.2 Series Connection of two Systems	94
3.4 Circular Interconnection	102
3.4.1 Circular Interconnection of SISO Systems	103
3.4.2 Feedback Interconnection	104
3.5 Partial State Reachability of a Series Connection of three Systems	106
3.6 General Networks	108
3.7 Homogeneous Networks	110
3.8 Conclusion	111
4 Applications to Coding Theory	113
4.1 Definitions and Basics about Block and Convolutional Codes	114
4.2 Correspondence between Linear Systems and Convolutional Codes	117
4.3 Probability of Non-Catastrophicity for a Convolutional Code	119
4.4 Interconnected Convolutional Codes	120
4.4.1 Series Connection of two Convolutional Codes	121
4.4.2 Parallel-Series Connection of two Convolutional Codes	122
4.4.3 Interleaved Parallel-Series Connection	126

4.4.4	Turbo Codes	126
4.5	Interconnection of Convolutional and Block Codes	130
4.5.1	Series Connection	131
4.5.2	Truncated Series Connection	135
4.6	Probability of MDP Convolutional Codes	140
4.6.1	Probability of MDP Codes with Rate $1/n$ and Degree 1	140
4.6.2	Probability Bounds for General MDP Convolutional Codes	141
4.7	Probability of MDS Block Codes	143
4.8	Random Linear Network Coding	143
4.8.1	Delay-free Acyclic Network Coding	144
4.8.2	Memory-free Convolutional Network Coding	147
4.9	Conclusion	150
Bibliography		151