

# Contents

## Preface — VII

### 1 Introduction — 1

- 1.1 Preliminaries — 1
- 1.2 Basic Concepts of the Ill-Posed Problems Theory — 14

### 2 Regularization Methods For Linear Equations — 20

- 2.1 Linear Equations in Hilbert Space — 21
- 2.2 Regularization Algorithms in Banach Space — 37
- 2.3 Necessity of Sourcewise Representation in Banach Space — 45
- 2.4 Ill-Posed Cauchy Problem in Banach Space — 74
- 2.5 Logarithmic Convergence of Solution Methods for Cauchy Problems — 77
- 2.6 Necessary Conditions for Logarithmic Estimates — 87
- 2.7 Linear Ill-Posed Problems Under Random Noise — 96

### 3 Finite Difference Methods — 101

- 3.1 Scalar Cauchy Problem — 102
- 3.2 Finite Difference Regularization Methods in Banach Space — 110
- 3.3 Power Rate of Convergence Estimates — 126
- 3.4 Necessity of the Source Condition — 134

### 4 Iterative Regularization Methods — 144

- 4.1 Parametric Approximation — 145
- 4.2 Regularized Gauss–Newton Methods — 153
- 4.3 Necessity of Sourcewise Representation in Hilbert Space — 163
- 4.4 Regularized Newton–Kantorovich Methods — 171
- 4.5 Necessity of Sourcewise Representation in Banach Space — 179
- 4.6 Iteratively Regularized Gradient Method — 184
- 4.7 Regularized Iterative Methods Under Large Noise — 191
- 4.8 Iteratively Regularized Methods Under Random Noise — 204
- 4.9 Convexity of Tikhonov Functional — 211

### 5 Finite-Dimensional Iterative Processes — 226

- 5.1 Adaptive Choice of Projectors — 227
- 5.2 A Priori Choice of Projectors — 239
- 5.3 Class of Stable Iterative Methods — 242
- 5.4 A Posteriori Estimates — 251
- 5.5 Conditionally Well-Posed Inverse Problems — 254

|          |  |
|----------|--|
| <b>6</b> | <b>Variational Inequalities and Optimization Problems — 269</b>  |
| 6.1      | Browder–Tikhonov Regularization — 269                            |
| 6.2      | Simple Iteration Method and Newton–Kantorovich Type Method — 277 |
| 6.3      | Iterative Regularization Methods for Noisy Data — 284            |
| 6.4      | Variational Inequalities on a Ball — 288                         |
| 6.5      | Tikhonov Regularization of Ill-Posed Optimization Problems — 296 |
| 6.6      | Purely Data-Driven Regularization Algorithms — 303               |

|                           |
|---------------------------|
| <b>Bibliography — 315</b> |
|---------------------------|

|                    |
|--------------------|
| <b>Index — 321</b> |
|--------------------|