

Contents

Preface — V

List of contributing authors — XIII

Part I: Modeling and discretization

Steffen Basting, Annalisa Quaini, Suncica Canic, and Roland Glowinski

1 On the implementation and benchmarking of an extended ALE method for FSI problems — 3

- 1.1 Introduction — 4**
- 1.2 Problem definition — 6**
 - 1.2.1 The fluid problem — 7**
 - 1.2.2 The structure problem — 7**
 - 1.2.3 The coupled fluid-structure interaction problem — 8**
- 1.3 Numerical representation of the geometry — 9**
 - 1.3.1 Optimal triangulations — 9**
 - 1.3.2 Interface aligned mesh — 11**
- 1.4 Partitioned methods for the fluid-structure interaction problem — 13**
 - 1.4.1 The Dirichlet–Neumann method — 14**
 - 1.4.2 The Robin–Neumann method — 15**
- 1.5 The fully discrete problem — 16**
 - 1.5.1 The discrete fluid subproblem — 16**
 - 1.5.2 The discrete structure problem — 18**
 - 1.5.3 Enforcement of the coupling conditions — 20**
- 1.6 Numerical results — 23**
 - 1.6.1 Validation of the structure solver — 23**
 - 1.6.2 Continuous vs. discontinuous pressure across the interface — 26**
 - 1.6.3 Moderate-amplitude oscillatory motion of the beam — 28**
 - 1.6.4 Large-amplitude oscillatory motion of the beam — 32**
- 1.7 Conclusions — 34**

Johan Hoffman, Bärbel Holm, and Thomas Richter

2 The locally adapted parametric finite element method for interface problems on triangular meshes — 41

- 2.1 Introduction — 41**
- 2.2 The locally adapted patch finite element method — 43**
 - 2.2.1 Definition of the finite element space on cut triangles — 44**

2.2.2	Quadrature on cut triangles —	45
2.2.3	Discrete variational formulation —	47
2.3	Maximum angle condition —	47
2.3.1	Two edges are cut —	49
2.3.2	A vertex and an edge are cut —	50
2.4	A priori error analysis —	52
2.5	Numerical examples —	57
2.5.1	Circular interface —	58
2.5.2	Horizontal cut —	59
2.5.3	Tilted interface line —	61
2.6	Conclusions —	63

Stefan Frei and Thomas Richter

3	An accurate Eulerian approach for fluid-structure interactions —	69
3.1	Introduction —	69
3.2	Modeling —	73
3.2.1	Continuum mechanics —	73
3.2.2	A monolithic fully Eulerian approach —	77
3.3	Comparison with other approaches —	79
3.3.1	Arbitrary Lagrangian–Eulerian method —	80
3.3.2	Euler–Lagrangian methods —	82
3.3.3	Immersed boundary methods —	85
3.4	Discretization —	86
3.4.1	Space discretization —	86
3.4.2	Pressure stabilization —	89
3.4.3	Temporal discretization —	91
3.4.4	Stabilization of the solid equation —	97
3.5	Validation by means of benchmark problems —	102
3.5.1	Setting of the benchmark problems —	102
3.5.2	Practical implementation —	103
3.5.3	FSI1 benchmark —	104
3.5.4	FSI3 benchmark —	105
3.6	A simple contact problem —	109
3.6.1	Example 1: Vertical fall —	109
3.6.2	A simple contact algorithm —	115
3.6.3	Example 2: Bouncing down the stairs —	117
3.7	Conclusion —	121

Part II: Solvers

Philipp Birken and Azahar Monge

4 Numerical methods for unsteady thermal fluid structure interaction — 129

- 4.1 Introduction — 129
 - 4.1.1 Partitioned coupling methods — 132
- 4.2 Treating thermal FSI problems — 134
 - 4.2.1 Fluid model — 134
 - 4.2.2 Structure model — 135
 - 4.2.3 Coupling conditions — 136
 - 4.2.4 Discretization in space — 136
 - 4.2.5 Time discretization — 136
 - 4.2.6 The Dirichlet–Neumann method — 137
- 4.3 A model problem: Coupled heat equations — 137
 - 4.3.1 Model problem — 138
 - 4.3.2 Semidiscrete analysis — 138
 - 4.3.3 Space discretization — 139
 - 4.3.4 Time discretization — 141
 - 4.3.5 Fixed-point iteration — 141
- 4.4 Convergence analysis — 142
 - 4.4.1 Numerical results — 146
- 4.5 Time-adaptive methods — 148
 - 4.5.1 Extrapolation from time integration — 150
- 4.6 Choosing tolerances — 151
 - 4.6.1 Perturbed nested fixed-point iteration — 152
 - 4.6.2 Application: Dirichlet–Neumann coupling for a transmission problem — 153
 - 4.6.3 Test case: Transmission problem — 156
- 4.7 Numerical results — 159
 - 4.7.1 Flow over a plate — 160
 - 4.7.2 Cooling of a flanged shaft — 161
- 4.8 Summary and conclusions — 164

Ulrich Langer and Huidong Yang

5 Recent development of robust monolithic fluid-structure interaction solvers — 169

- 5.1 Introduction — 169
- 5.2 Block factorization-based preconditioners — 171
 - 5.2.1 Direct LU decomposition — 171
 - 5.2.2 Block-triangular approximations — 172
 - 5.2.3 Combination with parallel preconditioners — 173
 - 5.2.4 The augmentation preconditioner — 175
 - 5.2.5 Block LU -type factorization — 176

5.2.6	A <i>LDU</i> block factorization —	177
5.3	Geometric multigrid methods —	179
5.3.1	A Vanka-like smoother —	179
5.3.2	A Dirichlet–Neumann smoother —	179
5.4	Algebraic multigrid methods —	180
5.4.1	Smoothed aggregation multigrid methods —	180
5.4.2	Matrix-graph-based algebraic multigrid methods —	181
5.5	Domain decomposition preconditioners —	182
5.5.1	A restricted additive Schwarz preconditioner —	182
5.5.2	A two-level Schwarz preconditioner —	182
5.6	Some numerical studies —	183
5.7	Conclusions —	186

Daniel Jodlbauer and Thomas Wick

6	A monolithic FSI solver applied to the FSI 1,2,3 benchmarks —	193
6.1	Introduction —	193
6.2	FSI equations —	195
6.2.1	Domains and boundaries —	195
6.2.2	Equations —	197
6.2.3	Discretization —	199
6.2.4	Summary —	207
6.3	Linear solvers —	209
6.3.1	Direct solvers —	209
6.3.2	Reordering schemes —	210
6.3.3	ILU —	210
6.3.4	Schur complement —	213
6.3.5	GMRES —	213
6.3.6	Multigrid methods —	215
6.4	Preconditioners —	217
6.4.1	Introduction —	218
6.4.2	First examples —	218
6.4.3	LU-approach —	220
6.4.4	Solving the subproblems —	223
6.4.5	Mesh —	223
6.4.6	Fluid —	223
6.4.7	Solid —	224
6.5	Numerical tests – FSI benchmarks —	224
6.5.1	Setup —	225
6.5.2	Test 1: Using direct solvers for the subproblems —	227
6.5.3	Test 2: Iterative subsolvers —	228
6.6	Conclusions and future work —	231
6.6.1	Conclusions —	231
6.6.2	Future work —	232

Part III: Applications

Claudia M. Colciago, Simone Deparis, and Davide Forti

7 Fluid-structure interaction for vascular flows: From supercomputers to laptops — 237

- 7.1 Introduction — 238
 - 7.1.1 Outline — 243
- 7.2 Modeling fluid-structure interaction in moving domains — 243
 - 7.2.1 Fluid-structure interaction modeling — 244
 - 7.2.2 Weak formulation — 246
 - 7.2.3 Fully implicit monolithic scheme — 247
 - 7.2.4 Algebraic formulation — 249
 - 7.2.5 Preconditioning by FaCSI — 250
- 7.3 Modeling reduction for FSI in arteries — 252
 - 7.3.1 Transpiration conditions — 252
 - 7.3.2 Thin-wall model — 253
 - 7.3.3 Weak formulation of the reduced fluid-structure interaction problem — 255
- 7.4 Reduced-order methods for reduced fluid-structure interaction models — 256
 - 7.4.1 Numerical reduction — 256
 - 7.4.2 Proper orthogonal decomposition — 257
- 7.5 Fluid-structure interaction in a patient-specific aortofemoral artery — 259
 - 7.5.1 Numerical results obtained by the full three-dimensional FSI model — 262
 - 7.5.2 Numerical simulation setting for the RFSI — 265
 - 7.5.3 Comparisons between FSI and RFSI — 271
- 7.6 Conclusions — 275

E. Harald van Brummelen, Mahnaz Shokrpour Roudbari, Görkem Şimşek, and Kristoffer G. van der Zee

8 Binary-fluid–solid interaction based on the Navier–Stokes–Cahn–Hilliard Equations — 283

- 8.1 Introduction — 283
- 8.2 Problem statement — 285
 - 8.2.1 Navier–Stokes–Cahn–Hilliard binary-fluid model — 286
 - 8.2.2 Hyperelastic St. Venant–Kirchhoff solid model — 290
 - 8.2.3 Interface conditions — 291
- 8.3 Weak formulation — 293
 - 8.3.1 ALE formulation of NSCH equations — 293
 - 8.3.2 Interface conditions — 295

8.3.3	Ambient function spaces —	297
8.3.4	Aggregated weak formulation —	301
8.4	Properties of the binary-fluid–solid interaction problem —	302
8.4.1	Energy dissipation —	302
8.4.2	Equilibrium solutions —	306
8.4.3	Equilibrium fluid–fluid interface profile —	307
8.5	Numerical approximation methods —	311
8.5.1	Spatial and temporal discretization —	311
8.5.2	Partitioned iterative solution method —	314
8.6	Numerical experiments —	316
8.6.1	Experimental set-up —	317
8.6.2	Numerical results —	318
8.7	Conclusion —	323

Thomas Wick

9	Coupling fluid-structure interaction with phase-field fracture: Algorithmic details —	329
9.1	Introduction —	329
9.1.1	Fluid-structure interaction (FSI) —	330
9.1.2	Phase-field fracture (PFF) —	331
9.1.3	Combining FSI and PFF —	335
9.2	Notation, spaces, variables, constitutive laws —	337
9.2.1	Philosophy —	337
9.2.2	Notation —	337
9.2.3	Function spaces —	338
9.2.4	Parameters, variables, vectors, tensors, and constitutive laws —	339
9.3	The fluid-structure interaction phase-field fracture (FSI-PFF) model —	340
9.3.1	The FSI-PFF system —	340
9.3.2	Treatment of the crack irreversibility constraint —	341
9.3.3	FSI interface coupling conditions —	341
9.3.4	Remarks on pressures —	342
9.4	Discretization —	342
9.4.1	Time stepping —	343
9.4.2	Adaptive time-step control with respect to goal functionals —	345
9.4.3	Spatial discretization —	346
9.4.4	Solution algorithms —	348
9.5	Numerical examples —	352
9.5.1	Example 1: Constant inflow —	354
9.5.2	Example 2: Sinus inflow —	355
9.6	Conclusions and future questions —	356

Index — 367