

Contents

Preface — IX

Acknowledgement — XI

List of Symbols — XVII

1 Vectors — 1

- 1.0 Introduction — 1
- 1.1 Vectors as ordered sets — 1
- 1.2 Geometry of vectors — 14
 - 1.2.1 Geometry of scalar multiplication — 14
 - 1.2.2 Geometry of addition of vectors — 14
 - 1.2.3 A coordinate-free definition of vectors — 15
 - 1.2.4 Geometry of dot products — 16
 - 1.2.5 Cauchy–Schwartz inequality — 17
 - 1.2.6 Orthogonal and orthonormal vectors — 19
 - 1.2.7 Projections — 21
 - 1.2.8 Work done — 24
- 1.3 Linear dependence and linear independence of vectors — 28
 - 1.3.1 A vector subspace — 38
 - 1.3.2 Gram–Schmidt orthogonalization process — 42
- 1.4 Some applications — 48
 - 1.4.1 Partial differential operators — 48
 - 1.4.2 Maxima/minima of a scalar function of many real scalar variables — 50
 - 1.4.3 Derivatives of linear and quadratic forms — 50
 - 1.4.4 Model building — 52

2 Matrices — 59

- 2.0 Introduction — 59
- 2.1 Various definitions — 60
 - 2.1.1 Some more practical situations — 75
- 2.2 More properties of matrices — 81
 - 2.2.1 Some more practical situations — 87
 - 2.2.2 Pre and post multiplications by diagonal matrices — 95
- 2.3 Elementary matrices and elementary operations — 100
 - 2.3.1 Premultiplication of a matrix by elementary matrices — 102
 - 2.3.2 Reduction of a square matrix into a diagonal form — 111
 - 2.3.3 Solving a system of linear equations — 113
- 2.4 Inverse, linear independence and ranks — 121

2.4.1	Inverse of a matrix by elementary operations —	121
2.4.2	Checking linear independence through elementary operations —	124
2.5	Row and column subspaces and null spaces —	128
2.5.1	The row and column subspaces —	129
2.5.2	Consistency of a system of linear equations —	133
2.6	Permutations and elementary operations on the right —	138
2.6.1	Permutations —	138
2.6.2	Postmultiplications by elementary matrices —	138
2.6.3	Reduction of quadratic forms to their canonical forms —	145
2.6.4	Rotations —	147
2.6.5	Linear transformations —	148
2.6.6	Orthogonal bases for a vector subspace —	152
2.6.7	A vector subspace, a more general definition —	154
2.6.8	A linear transformation, a more general definition —	156
2.7	Partitioning of matrices —	160
2.7.1	Partitioning and products —	161
2.7.2	Partitioning of quadratic forms —	164
2.7.3	Partitioning of bilinear forms —	165
2.7.4	Inverses of partitioned matrices —	166
2.7.5	Regression analysis —	170
2.7.6	Design of experiments —	172
3	Determinants —	181
3.0	Introduction —	181
3.1	Definition of the determinant of a square matrix —	181
3.1.1	Some general properties —	183
3.1.2	A mechanical way of evaluating a 3×3 determinant —	189
3.1.3	Diagonal and triangular block matrices —	195
3.2	Cofactor expansions —	203
3.2.1	Cofactors and minors —	203
3.2.2	Inverse of a matrix in terms of the cofactor matrix —	208
3.2.3	A matrix differential operator —	211
3.2.4	Products and square roots —	215
3.2.5	Cramer's rule for solving systems of linear equations —	216
3.3	Some practical situations —	223
3.3.1	Cross product —	223
3.3.2	Areas and volumes —	225
3.3.3	Jacobians of transformations —	229
3.3.4	Functions of matrix argument —	239
3.3.5	Partitioned determinants and multiple correlation coefficient —	241
3.3.6	Maxima/minima problems —	245

4	Eigenvalues and eigenvectors — 253
4.0	Introduction — 253
4.1	Eigenvalues of special matrices — 253
4.2	Eigenvectors — 260
4.2.1	Some definitions and examples — 260
4.2.2	Eigenvalues of powers of a matrix — 267
4.2.3	Eigenvalues and eigenvectors of real symmetric matrices — 269
4.3	Some properties of complex numbers and matrices in the complex fields — 280
4.3.1	Complex numbers — 280
4.3.2	Geometry of complex numbers — 281
4.3.3	Algebra of complex numbers — 283
4.3.4	n -th roots of unity — 286
4.3.5	Vectors with complex elements — 289
4.3.6	Matrices with complex elements — 291
4.4	More properties of matrices in the complex field — 298
4.4.1	Eigenvalues of symmetric and Hermitian matrices — 298
4.4.2	Definiteness of matrices — 307
4.4.3	Commutative matrices — 310
5	Some applications of matrices and determinants — 325
5.0	Introduction — 325
5.1	Difference and differential equations — 325
5.1.1	Fibonacci sequence and difference equations — 325
5.1.2	Population growth — 331
5.1.3	Differential equations and their solutions — 332
5.2	Jacobians of matrix transformations and functions of matrix argument — 341
5.2.1	Jacobians of matrix transformations — 342
5.2.2	Functions of matrix argument — 348
5.3	Some topics from statistics — 354
5.3.1	Principal components analysis — 354
5.3.2	Regression analysis and model building — 358
5.3.3	Design type models — 362
5.3.4	Canonical correlation analysis — 364
5.4	Probability measures and Markov processes — 371
5.4.1	Invariance of probability measures — 372
5.4.2	Discrete time Markov processes and transition probabilities — 374
5.5	Maxima/minima problems — 381
5.5.1	Taylor series — 382
5.5.2	Optimization of quadratic forms — 387

5.5.3	Optimization of a quadratic form with quadratic form constraints —	389
5.5.4	Optimization of a quadratic form with linear constraints —	390
5.5.5	Optimization of bilinear forms with quadratic constraints —	392
5.6	Linear programming and nonlinear least squares —	398
5.6.1	The simplex method —	400
5.6.2	Nonlinear least squares —	406
5.6.3	Marquardt's method —	408
5.6.4	Mathai–Katiyar procedure —	410
5.7	A list of some more problems from physical, engineering and social sciences —	411
5.7.1	Turbulent flow of a viscous fluid —	411
5.7.2	Compressible flow of viscous fluids —	412
5.7.3	Heat loss in a steel rod —	412
5.7.4	Small oscillations —	413
5.7.5	Input–output analysis —	414
6	Matrix series and additional properties of matrices —	417
6.0	Introduction —	417
6.1	Matrix polynomials —	417
6.1.1	Lagrange interpolating polynomial —	418
6.1.2	A spectral decomposition of a matrix —	420
6.1.3	An application in statistics —	422
6.2	Matrix sequences and matrix series —	424
6.2.1	Matrix sequences —	424
6.2.2	Matrix series —	426
6.2.3	Matrix hypergeometric series —	429
6.2.4	The norm of a matrix —	430
6.2.5	Compatible norms —	434
6.2.6	Matrix power series and rate of convergence —	435
6.2.7	An application in statistics —	435
6.3	Singular value decomposition of a matrix —	438
6.3.1	A singular value decomposition —	440
6.3.2	Canonical form of a bilinear form —	443

References —	447
---------------------	------------

Index —	449
----------------	------------