

Contents

1	Introduction	1
1.1	Fundamental Physical Laws	1
1.1.1	Cartesian Coordinates	1
1.1.2	The Navier-Stokes Equations	1
1.1.3	Boundary Fluxes	3
1.1.4	The Hydrostatic Approximation	3
1.1.5	The Stability Frequency	4
1.2	Numerical Methods	4
1.2.1	Finite Differences	4
1.2.2	Requirements for a Finite-Difference Model	5
1.3	Modelling with FORTRAN 95	5
1.3.1	Writing and Compiling Codes	5
1.3.2	Modular Source Codes	6
1.4	Visualisation with SciLab	6
1.4.1	Writing SciLab Scripts	6
1.4.2	GIF Animations	7
1.5	Organisation of Work	8
1.6	Download of Computer Codes	8
2	1D Models of Ekman Layers	9
2.1	Useful Background Knowledge	9
2.1.1	Inertial Oscillations	9
2.1.2	Semi-implicit Treatment of the Coriolis Force	10
2.2	The Surface Ekman Layer	11
2.2.1	Boundary-Layer Equations	11
2.2.2	Scaling: The Temporal Rossby Number	11
2.2.3	Scaling: The Ekman Number	12
2.2.4	Solutions of the Boundary-Layer Equations	13
2.2.5	Finite-Difference Equations	13
2.2.6	Formulation of Diffusion Terms	14
2.2.7	Stability Criterion for Diffusion Terms	14

2.3	Exercise 1: The Surface Ekman Layer	15
2.3.1	Task Description	15
2.3.2	Results	16
2.3.3	Explanation of the Ekman-Layer Structure	17
2.3.4	Additional Exercises for the Reader	17
2.4	The Bottom Ekman Layer	18
2.4.1	Boundary-Layer Equations	18
2.5	Exercise 2: The Bottom Ekman Layer	18
2.5.1	Task Description	18
2.5.2	Results	18
2.5.3	Additional Exercises for the Reader	19
3	Basics of Nonhydrostatic Modelling	21
3.1	Level Models	21
3.2	2D Vertical-Slice Modelling	22
3.2.1	Configuration	22
3.2.2	The Arakawa C-Grid	23
3.3	Surface Gravity Waves	24
3.3.1	The Governing Equations	24
3.3.2	The Dispersion Relation	24
3.3.3	Orbital Motions of Water Particles and Wave Pressure	26
3.4	Nonhydrostatic Solver	26
3.4.1	Splitting Pressure into Parts	26
3.4.2	Starting as Simple as Possible	27
3.4.3	Finite-Difference Scheme	27
3.4.4	The S.O.R. Method	29
3.4.5	Boundary Conditions for Variable Bathymetry	31
3.4.6	Stability Criterion	31
3.5	Exercise 3: Short Surface Gravity Waves	32
3.5.1	Aim	32
3.5.2	Task Description	32
3.5.3	Results	32
3.5.4	Additional Exercise for the Reader	33
3.5.5	Implementation of Variable Bottom Topography	34
3.5.6	Results	35
3.6	Inclusion of Variable Density	35
3.6.1	The Governing Equations	35
3.6.2	Discretisation of the Advection Terms	36
3.6.3	Stability Criterion for the Advection Equation	38
3.6.4	Implementation of Density Diffusion	38
3.6.5	Required Modifications of the Code	39
3.7	Exercise 4: Density-Driven Flows	40
3.7.1	Aim	40
3.7.2	Task Description	40

3.7.3	Theory	41
3.7.4	Results	41
3.7.5	Can Reduced-Gravity Plumes Jump?	41
3.7.6	Additional Exercise for the Reader	43
3.7.7	The Rigid-Lid Approximation	43
3.8	Internal Waves	44
3.8.1	Theory	44
3.8.2	Normal Wave Modes	45
3.9	Exercise 5: Internal Waves	46
3.9.1	Aim	46
3.9.2	Task Description	46
3.9.3	Results	47
3.9.4	Additional Exercise for the Reader	48
3.10	Mechanical Turbulence	48
3.10.1	Kelvin-Helmholtz Instability	48
3.10.2	Instability of a Stratified Shear Flow	49
3.11	Exercise 6: Kelvin-Helmholtz Instability	50
3.11.1	Aim	50
3.11.2	Task Description	51
3.11.3	Cyclic Boundary Conditions	51
3.11.4	Results	52
3.11.5	Additional Exercise for the Reader	53
3.12	Lee Waves and the Froude Number	53
3.12.1	The Hydraulic Jump	53
3.13	Exercise 7: Lee Waves	54
3.13.1	Task Description	54
3.13.2	Results: Continuous Density Stratification	55
3.13.3	Results: Two-Layer Stratification	56
3.13.4	Additional Exercise for the Reader	57
3.14	Oceanic Convection	57
3.14.1	Background	57
3.14.2	Free Convection	57
3.14.3	The Flux-Rayleigh Number	58
3.14.4	Aspect Ratio of Convection Cells	59
3.14.5	Convective Mixed-Layer Deepening	59
3.15	Exercise 8: Free Convection	60
3.15.1	Aim	60
3.15.2	Task Description	61
3.15.3	A Trick to Avoid Substantial Round-off Errors	62
3.15.4	Inclusion of Momentum Diffusion and Bottom Friction ...	62
3.15.5	Results	64
3.15.6	Additional Exercise for the Reader	65
3.16	Exercise 9: Convective Entrainment	65
3.16.1	How It Works	65
3.16.2	Entrainment Velocity	65

3.16.3	Task Description	66
3.16.4	Results	66
3.16.5	Additional Exercises for the Reader	67
3.17	Exercise 10: Slope Convection near the Shore	67
3.17.1	Background	67
3.17.2	Implementation of Bottom Friction on a Sloping Terrain ..	68
3.17.3	Task Description	68
3.17.4	Results	70
3.17.5	Additional Exercise for the Reader	71
3.18	Double Diffusion	72
3.18.1	Background	72
3.18.2	Double-Diffusive Instability	72
3.18.3	Double-Diffusive Layering	73
3.18.4	The Gradient Ratio and the Turner Angle	73
3.19	Exercise 11: Double-Diffusive Instability	74
3.19.1	Aim	74
3.19.2	Task Description	74
3.19.3	Results	75
3.20	Exercise 12: Double-Diffusive Layering	77
3.20.1	Aim	77
3.20.2	Task Description	77
3.20.3	Results	78
3.20.4	Additional Exercises for the Reader	79
3.21	Tilted Coordinate Systems	79
3.21.1	The Governing Equations	79
3.22	Exercise 13: Stratified Flows on a Slope	81
3.22.1	Aim	81
3.22.2	Task Description	81
3.22.3	Results	82
3.22.4	Additional Exercise for the Reader	83
3.23	Estuaries	83
3.23.1	Definition	83
3.23.2	Classification of Estuaries According to Origin	84
3.23.3	The Dynamics of Positive Estuaries	84
3.23.4	Brief Overview of Tides	84
3.23.5	Dynamic Theory of Tides	85
3.23.6	Tides in Estuaries	85
3.23.7	Tidal Patterns	86
3.23.8	Classification of Estuaries According to Stratification and Circulation	86
3.23.9	Transport Timescales in Estuaries	87
3.24	Exercise 14: Positive Estuaries	89
3.24.1	Aim	89
3.24.2	Task Description	89
3.24.3	Implementation of Variable Channel Width	91

3.24.4	Advanced Turbulence Closure	91
3.24.5	Results	92
3.24.6	Additional Exercises for the Reader	93
3.25	Exercise 15: Inverse Estuaries	94
3.25.1	Aim	94
3.25.2	Task Description	94
3.25.3	Results	95
3.25.4	Additional Exercise for the Reader	96
4	2.5D Vertical Slice Modelling	97
4.1	The Basis	97
4.1.1	Adding Another Half Dimension	97
4.1.2	The Geostrophic Balance	97
4.1.3	Scaling	99
4.1.4	Conservation of Potential Vorticity	99
4.1.5	Geostrophic Adjustment	100
4.1.6	The 2.5d Shallow-Water Model	101
4.1.7	Implementation of the Coriolis Force	101
4.1.8	Potential Problems	102
4.2	Exercise 16: Geostrophic Adjustment	103
4.2.1	Aim	103
4.2.2	Task Description	103
4.2.3	Results	104
4.2.4	Additional Exercise for the Reader	105
4.3	Exercise 17: Tidal-Mixing Fronts	106
4.3.1	Background	106
4.3.2	Task Description	106
4.3.3	Results	107
4.3.4	Additional Study	109
4.3.5	Results and Discussion	109
4.3.6	Additional Exercises for the Reader	110
4.4	Coastal Upwelling	110
4.4.1	Background	110
4.4.2	How Does It Work?	111
4.4.3	Partial and Full Upwelling	111
4.4.4	The Upwelling Index	113
4.5	Exercise 18: Coastal Upwelling and Downwelling	113
4.5.1	Aim	113
4.5.2	Task Description	113
4.5.3	Advanced Turbulence Closure	114
4.5.4	Results: Upwelling Scenario	115
4.5.5	Additional Exercise for the Reader	116
4.5.6	Results: Downwelling Scenario	116
4.5.7	Additional Exercise for the Reader	117

4.6	Exercise 19: Ekman Pumping	118
4.6.1	Theoretical Background.....	118
4.6.2	Aim	118
4.6.3	Task Description	118
4.6.4	Results: Scenario 1	120
4.6.5	Results: Scenario 2	122
4.6.6	Results: Scenario 3	123
4.6.7	Additional Exercises for the Reader	124
5	3D Level Modelling	125
5.1	The Basic Equations	125
5.1.1	The Basics	125
5.1.2	Conservation of Momentum	125
5.1.3	Conservation of Volume	126
5.1.4	Evolution of the Density Field	127
5.2	Numerical Treatment.....	127
5.2.1	The 3d Arakawa C-grid	127
5.2.2	Treatment of the Advection Terms	128
5.2.3	The Nonhydrostatic Solver of the Momentum Equations ..	129
5.2.4	Stability Criteria	130
5.3	Exercise 20: Geostrophic Adjustment in 3D.....	131
5.3.1	Aim	131
5.3.2	Task Description	131
5.3.3	Results	132
5.3.4	Additional Exercise for the Reader	132
5.4	Exercise 21: Eddy Formation in a Strait	133
5.4.1	Background	133
5.4.2	Aim	134
5.4.3	Task Description	134
5.4.4	Creation of Variable Bathymetry	136
5.4.5	Results	136
5.4.6	Bathymetry Creation	137
5.4.7	Additional Exercises for the Reader	137
5.5	Exercise 22: Exchange Flow Through a Strait	137
5.5.1	Aim	137
5.5.2	Mediterranean Seas	138
5.5.3	Task Description	139
5.5.4	Results	140
5.5.5	Additional Exercise for the Reader	142
5.6	Exercise 23: Coastal Upwelling in 3D.....	142
5.6.1	Aim	142
5.6.2	Task Description	142
5.6.3	Results	143
5.6.4	Additional Exercise for the Reader	145
5.6.5	Time-Splitting Methods	145

5.7	The Thermohaline Circulation	146
5.7.1	The Abyssal Circulation	146
5.7.2	The Stommel-Arons Model	146
5.8	Exercise 24: The Abyssal Circulation	148
5.8.1	Aim	148
5.8.2	Task Description	148
5.8.3	Results	150
5.8.4	Additional Exercise for the Reader	152
5.8.5	Improved Float Tracking	152
5.9	The Equatorial Barrier	155
5.9.1	Inertial Oscillations About the Equator	155
5.9.2	Variation to Exercise 24	156
5.9.3	Results	156
5.9.4	Additional Exercise for the Reader	157
5.10	Equatorial Waves	158
5.10.1	Background	158
5.10.2	Equatorial Kelvin Waves	158
5.10.3	Other Equatorially Trapped Waves	159
5.11	The El-Niño Southern Oscillation	161
5.11.1	Background	161
5.12	Exercise 25: Simulation of an El-Niño Event	162
5.12.1	Aim	162
5.12.2	Task Description	162
5.12.3	The Smagorinsky Turbulence Closure Scheme	164
5.12.4	Warning	164
5.12.5	Results	164
5.12.6	Additional Exercises for the Reader	165
5.13	Advanced Lateral Boundary Conditions	166
5.13.1	Background	166
5.13.2	Consistency	166
5.13.3	Inflow Conditions	166
5.13.4	Outflow Conditions	167
5.13.5	Zero-Gradient Conditions	168
5.13.6	Radiation Conditions	169
5.13.7	Sponge Layers and Low-Pass Grid Filters	170
5.14	Final Remark	171
5.15	Technical Information	171
	Bibliography	173
	List of Exercises	177
	Index	179