

Contents

List of Figures	XI
List of Tables	XVI
Notation and Symbols	XVII
1 Introduction	1
1.1 Outline of the Thesis	4
1.2 Notation	5
2 WEB-Splines	7
2.1 B-Splines	7
2.2 Weight Functions	12
2.2.1 Construction of Weight Functions	13
2.3 WEB-Splines	16
2.3.1 Marsden's Identity	17
2.3.2 Extension of B-Splines	18
2.4 Hierarchical WEB-Splines	21
2.4.1 Hierarchical Splines	21
2.4.2 Extension of a Hierarchical B-Spline Basis	26
2.4.3 Overview	34
2.5 Dual Functions	34
3 Hierarchical Quasi-Interpolant	37
3.1 Polynomial Approximation	38
3.2 Quasi-Interpolants	39
3.2.1 Example 1: Quasi-Interpolant for L^2 -Functions	40

3.2.2	Example 2: Schoenberg's Scheme	41
3.3	Hierarchical Quasi-Interpolant	43
3.3.1	Projection	46
3.3.2	Boundedness	47
3.4	Error Estimation	50
3.5	Asymptotic Optimal Grids	52
3.5.1	$\Omega = [0, 1]$ and $p = \infty$	52
3.5.2	$\Omega = [0, 1]$ and $p = 2$	59
3.5.3	$\Omega = [0, 1]^2$ and $p = 2$	62
3.5.4	L-Shaped Domain for $p = 2$	67
4	Adaptive Finite Element Method with Weighted B-Splines	69
4.1	Basic Principles	70
4.1.1	Abstract Variational Problem	70
4.1.2	Finite Element Method with Weighted Splines	74
4.2	A-Posteriori Error Estimation	77
4.3	Adaptive Algorithm	85
5	Adaptive WEB-Collocation	91
5.1	Uniform WEB-Collocation	92
5.2	Hierarchical WEB-Collocation	95
5.3	Adaptive Collocation Algorithm	100
6	Elliptic Problems with Singular Solutions	105
6.1	Laplace Equation	106
6.1.1	Numerical Results	108
6.2	Linear Elasticity	117
6.2.1	Lamé-Navier Equations	117
6.2.2	Plane Strain and Plane Stress	120
6.2.3	Singular Solutions	123
6.2.4	Numerical Approximation with B-Splines	127
6.2.5	Numerical Results	128
6.3	Conclusion	134
7	Time-Dependent Problems with WEB-Collocation	137
7.1	Heat Equation	138

7.1.1	Basic Model	138
7.1.2	Example	141
7.2	Tsunami Modeling with the Wave Equation	144
7.2.1	Shallow Water Equations	144
7.2.2	Approximation	148
7.2.3	Example	149
8	Summary	155
	Bibliography	159