

Contents

1	Introduction	15
I	Some general results	25
2	Point Processes and Coincidences	27
2.1	Introduction	27
2.2	Regular processes and coincidence densities	28
2.2.1	Assumptions for orderly processes.	28
2.2.2	Coincidence and exclusion probabilities.	30
2.3	Shot noise processes	34
2.3.1	Definition	34
2.3.2	Characteristic function of order q	35
2.3.3	Joint distribution of counting variables	37
2.4	Exclusion densities	41
2.4.1	Exclusion (probability) densities	41
2.4.2	Distances between instants and impulses	42
2.4.3	Intervals between neighbouring impulses	43
2.4.4	Survivaltime and lifetime	44

2.5	Stationarity	45
2.6	Operations on point processes	47
2.6.1	Superposition	47
2.6.2	Random thinning	49
2.7	Examples	49
2.7.1	The Poisson process	49
2.7.2	Detection of a mono-mode field with n photons	51
2.A	Additional proofs	54
3	Laws of regular point processes	57
3.1	The space of realizations	57
3.1.1	Symmetric sets in $\mathcal{X}^n$	58
3.2	Events	58
3.2.1	The counting algebra	59
3.2.2	Distributions of p elements	62
3.3	Probability laws	67
3.3.1	Exclusion probabilities.	68
3.3.2	Symmetry of laws	69
3.3.3	Coincidence probabilities	71
3.3.4	Probabilities of cylinders in $\mathcal{C}_2^p$ and $\mathcal{C}_2$	74
3.3.5	Moments and factorial moments.	75
3.4	Regular point processes.	80
3.4.1	Densities of exclusion probabilities	80
3.4.2	Coincidence probability densities	82
3.4.3	Construction of point processes	87
3.5	σ -finite point processes	91
3.5.1	Exclusion probabilities.	92
3.5.2	Coincidence densities	93
3.6	Characteristic functionals	96

II	Point process models	101
4	Cox processes and photoelectrons	103
4.1	Introduction	103
4.2	Detection of an optical field	104
4.3	The Cox process	106
4.3.1	Definition	106
4.3.2	Characteristic and generating function	108
4.3.3	Exclusion probabilities.	110
4.3.4	Stationarity	112
4.3.5	Superposition and thinning	112
4.4	Thermal Light	113
4.4.1	Introduction	113
4.4.2	The generating function of impulses	114
4.4.3	The method of coincidences	115
4.4.4	Exclusion probabilities.	116
4.4.5	Example: Thermal light with Lorentz-spectrum	119
4.5	The doubly stochastic Poisson process	122
4.5.1	Joint laws.	122
4.5.2	Coincidences	123
4.6	The Gauss-Poisson process	125
4.6.1	Motivation	125
4.6.2	The characteristic functional	125
4.6.3	Coincidences	129
4.6.4	Exclusion probabilities and an Existence theorem	130
4.6.5	The stationary case.	132
4.6.6	An example	133
4.6.7	Proof of Theorem 4.4	134
4.A	Proof of Formula (4.66)	135
5	Renewal processes	139
5.1	Definition	139
5.2	Exclusion probabilities.	140
5.3	Joint counting laws	141

6	Fermions in a chaotic state	145
6.1	Introduction	145
6.2	Characterization of Fermions.	146
6.2.1	Generalities.	146
6.2.2	Generating function of the counting variables	147
6.2.3	Necessary and sufficient conditions for g to be a gener- ating function	150
6.2.4	A basic Theorem.	155
6.3	Fermions in a chaotic state: Physical point of view	162
6.3.1	Introduction	162
6.3.2	The modes	162
6.3.3	Density matrices. Emission and detection processes	163
6.3.4	Discussion	165
6.4	An analogy between chaotic beams of fermions and of bosons	168
6.5	An example	170
6.5.1	Introduction	170
6.5.2	$\mathcal{P}_t$ is a generalized renewal process	170
6.5.3	Lorentzian assumptions	172
III	Optical Communications	175
7	Weak optical signals	177
7.1	Introduction	177
7.2	Estimation of a modulated optical signal	178
7.2.1	The problem	178
7.2.2	General results	179
7.2.3	The case of weak noise	182
7.2.4	An example	184
7.3	Estimation of the intensity of thermal light	188
7.4	Linear estimation	193
7.4.1	Internal noise of the detector	194
7.4.2	Noise from parasitical radiation	196

7.5	Detection problems	199
7.5.1	Thermal noise of the detector	200
7.5.2	Parasitical environmental radiation	203
7.5.3	Final remarks	205
7.6	Conclusions	206
7.A	Some general properties of the estimators.. . . .	207
7.A.1	Expectation of $a_q\{t_n\}$	207
7.A.2	Variance of $a_q\{t_n\}$	208
7.A.3	Proof of Formula (7.19)	209
7.B	The resolvent	210

IV Later mathematical developments 215

8 Quantum-classical processes 217

8.1	Introduction	218
8.1.1	Some mathematical tools	223
8.2	A general construction scheme	225
8.2.1	Nehring's representation of $\mathfrak{S}_L$	227
8.3	A special class of functionals L	229
8.3.1	Global assumptions on the potentials.	234
8.3.2	Infinitely extended processes	237
8.3.3	Explicit conditions for the potentials	239
8.4	The cluster equation	240
8.4.1	A general principle implying a Cox structure	242
8.5	Factorial moment measures	244
8.6	A representation of the modified Laplace transform	246

9 Quantum processes 247

9.1	Construction of quantum processes.	247
9.1.1	A construction method inducing quantum interactions	249
9.2	Immanantal processes	250
9.3	Cox processes	253
9.4	Support properties	254

9.5	Gibbsian character	255
9.6	Invariance properties and laws of large numbers	256
9.7	Mixing of quantum processes	259
9.8	A central limit theorem for quantum processes	262
10	Examples of quantum processes	265
10.1	Quantum Cox processes	267
10.1.1	Thermal light. The photon process.	267
10.2	Quantum Pólya processes	269
10.2.1	$\mathfrak{P}^\varepsilon$ is a Papangelou process	270
10.3	Ideal Bosons and Fermions	272
10.3.1	Laws of large numbers and central limit theorems.	273
10.3.2	Clustering representation of $\mathfrak{G}^+$	274
10.4	Quantum renewal processes	275
10.5	Ginibre processes	276
10.5.1	The exponential kernel is a quantum interaction	276
10.5.2	Construction	278
10.5.3	Gibbsian and Coxian character	279
10.5.4	Asymptotic behaviour	279
11	Pressure of quantum systems	283
11.1	The problem	283
11.1.1	A decomposition	284
11.1.2	The second term	285
11.2	Examples	286
11.2.1	Pólya processes	286
11.2.2	Ginibre processes	286
11.2.3	Ideal Bosons and Fermions in the complex plane	288
	Bibliography	291