

Contents

1	Preliminaries — 1
1.1	Preliminaries in probability — 1
1.1.1	Probability space — 1
1.1.2	Random variable and probability distribution — 5
1.1.3	Mathematical expectation and momentum — 8
1.2	Some preliminaries of stochastic process — 13
1.2.1	Markov process — 15
1.2.2	Preliminaries on ergodic theory — 22
1.3	Martingale — 25
1.4	Wiener process and Brown motion — 33
1.5	Poisson process — 39
1.6	Lévy process — 41
1.6.1	Characteristic function and infinite divisibility — 41
1.6.2	Lévy process — 42
1.6.3	Lévy-Itô decomposition — 44
1.7	The fractional Brownian motion — 47
2	The stochastic integral and Itô formula — 49
2.1	Stochastic integral — 49
2.1.1	Itô integral — 50
2.1.2	The stochastic integral in general case — 53
2.1.3	Poisson stochastic integral — 55
2.2	Itô formula — 56
2.3	The infinite-dimensional case — 61
2.3.1	Q-Wiener process and the stochastic integral — 61
2.3.2	Itô formula — 65
2.4	Nuclear operator and HS operator — 67
3	OU processes and SDEs — 70
3.1	Ornstein-Uhlenbeck processes — 70
3.2	Linear SDEs — 74
3.3	Nonlinear SDEs — 79
4	Random attractors — 82
4.1	Determinate nonautonomous systems — 82
4.2	Stochastic dynamical systems — 84
5	Applications — 88
5.1	Stochastic GL equation — 88
5.1.1	The existence of random attractor — 90
5.1.2	Hausdorff dimension of random attractor — 94
5.1.3	Generalized SGLE — 100

5.2	Ergodicity for SGL with degenerate noise — 101
5.2.1	Momentum estimate and pathwise uniqueness — 104
5.2.2	Invariant measures — 111
5.2.3	Ergodicity — 115
5.2.4	Some remarks — 129
5.3	Stochastic damped forced Ostrovsky equation — 129
5.3.1	Introduction — 129
5.3.2	Well-posedness — 131
5.3.3	Uniform estimates of solutions — 141
5.3.4	Asymptotic compactness and random attractors — 150
5.4	Simplified quasi-geostrophic model — 153
5.4.1	The existence and uniqueness of solution — 155
5.4.2	Existence of random attractors — 161
5.5	Stochastic primitive equations — 167
5.5.1	Stochastic 2D primitive equations with Lévy noise — 168
5.5.2	Large deviation for stochastic primitive equations — 188

Bibliography — 207

Index — 219