

Contents

Part I Mathematics of Distances

1	General Definitions	3
1.1	Basic Definitions	3
1.2	Main Distance-Related Notions	12
1.3	Metric Numerical Invariants	23
1.4	Main Mappings of Metric Spaces	36
1.5	General Distances	46
2	Topological Spaces	63
3	Generalizations of Metric Spaces	71
3.1	m -Tuple Generalizations of Metrics	71
3.2	Indefinite Metrics	73
3.3	Topological Generalizations	75
3.4	Beyond Numbers	78
4	Metric Transforms	87
4.1	Metrics on the Same Set	87
4.2	Metrics on Set Extensions	91
4.3	Metrics on Other Sets	94
5	Metrics on Normed Structures	97

Part II Geometry and Distances

6	Distances in Geometry	111
6.1	Geodesic Geometry	111
6.2	Projective Geometry	117
6.3	Affine Geometry	123
6.4	Non-Euclidean Geometry	125

7 Riemannian and Hermitian Metrics..... 135

7.1 Riemannian Metrics and Generalizations 136

7.2 Riemannian Metrics in Information Theory 152

7.3 Hermitian Metrics and Generalizations 156

8 Distances on Surfaces and Knots 169

8.1 General Surface Metrics 169

8.2 Intrinsic Metrics on Surfaces 175

8.3 Distances on Knots 180

9 Distances on Convex Bodies, Cones, and Simplicial Complexes 183

9.1 Distances on Convex Bodies..... 183

9.2 Distances on Cones..... 189

9.3 Distances on Simplicial Complexes 192

Part III Distances in Classical Mathematics

10 Distances in Algebra..... 199

10.1 Group Metrics 199

10.2 Metrics on Binary Relations 209

10.3 Metrics on Semilattices 210

11 Distances on Strings and Permutations 215

11.1 Distances on General Strings 216

11.2 Distances on Permutations 225

12 Distances on Numbers, Polynomials, and Matrices..... 229

12.1 Metrics on Numbers..... 229

12.2 Metrics on Polynomials 235

12.3 Metrics on Matrices 237

13 Distances in Functional Analysis..... 247

13.1 Metrics on Function Spaces..... 247

13.2 Metrics on Linear Operators 254

14 Distances in Probability Theory 259

14.1 Distances on Random Variables 260

14.2 Distances on Distribution Laws 261

Part IV Distances in Applied Mathematics

15 Distances in Graph Theory 277

15.1 Distances on the Vertices of a Graph 278

15.2 Distance-Defined Graphs 286

15.3 Distances on Graphs 297

15.4 Distances on Trees 305

16 Distances in Coding Theory	313
16.1 Minimum Distance and Relatives	315
16.2 Main Coding Distances	317
17 Distances and Similarities in Data Analysis	327
17.1 Similarities and Distances for Numerical Data	329
17.2 Relatives of Euclidean Distance	333
17.3 Similarities and Distances for Binary Data	337
17.4 Correlation Similarities and Distances	340
18 Distances in Systems and Mathematical Engineering	347
18.1 Distances in State Transition and Dynamical Systems	348
18.2 Distances in Control Theory	354
18.3 Motion Planning Distances	356
18.4 MOEA Distances	361

Part V Computer-Related Distances

19 Distances on Real and Digital Planes	365
19.1 Metrics on Real Plane	365
19.2 Digital Metrics	376
20 Voronoi Diagram Distances	383
20.1 Classical Voronoi Generation Distances	384
20.2 Plane Voronoi Generation Distances	386
20.3 Other Voronoi Generation Distances	390
21 Image and Audio Distances	393
21.1 Image Distances	393
21.2 Audio Distances	409
22 Distances in Networks	419
22.1 Scale-Free Networks	419
22.2 Network-Based Semantic Distances	424
22.3 Distances in Internet and Web	428

Part VI Distances in Natural Sciences

23 Distances in Biology	439
23.1 Genetic Distances	443
23.2 Distances for DNA/RNA and Protein Data	455
23.3 Distances in Ecology, Biogeography, Ethology	467
23.4 Other Biological Distances	484
24 Distances in Physics and Chemistry	499
24.1 Distances in Physics	499
24.2 Distances in Chemistry and Crystallography	524

25 Distances in Earth Science and Astronomy 533

25.1 Distances in Geography 533

25.2 Distances in Geophysics 546

25.3 Distances in Astronomy 558

26 Distances in Cosmology and Theory of Relativity 577

26.1 Distances in Cosmology 577

26.2 Distances in Theory of Relativity 587

Part VII Real-World Distances

27 Length Measures and Scales 613

27.1 Length Scales 613

27.2 Orders of Magnitude for Length 623

28 Distances in Applied Social Sciences 629

28.1 Distances in Perception and Psychology 629

28.2 Distances in Economics and Human Geography 640

28.3 Distances in Sociology and Language 653

28.4 Distances in Philosophy, Religion and Art 664

29 Other Distances 683

29.1 Distances in Medicine, Anthropometry and Sport 683

29.2 Equipment Distances 701

29.3 Miscellany 716

References 723

Index 731