Contents

In	troduction	1 [1]
First Part: On Functions of a Real Variable and Their Representation in an Orthogonal Coordinate System		3 [2]
I.	Remarks on the Single Independent Variable x	5 [2]
	Empirical and Abstract Precision. The Modern Concept of Number	5 [2]
	also in Pure Geometry	7 [4]
	Different Examples from Geometry Explanation Through Two Simple Propositions on Point Sets	10 [8] 12 [10]
II.	Functions $f(x)$ of a Real Variable x	15 [13]
	(Idea of the Function Stripe)	15 [13]
	On the Efficiency of Space Intuition	18 [17]
	(with a Digression About Differing Conceptions Regarding the Constitution of Matter)	19 [18]
	Curvature	22 [22]
	is the Analogy with an Empirical Curve?	27 [26]
	The Integrability of Continuous Functions	31 [30]
	and of the Smallest Value	35 [33]
	The Four Derivates	37 [35]

xvi Contents

	Weierstraß's Non-Differentiable Function.		
	Its General Characteristics	41	[39]
	Its Non-Differentiability		[45]
	The "Reasonable" Functions		[50]
III.		55	[51]
	Approximation of Empirical Curves		
	by Means of Reasonable Functions	55	[51]
	Approximation of a Reasonable Function		
	by Means of Simple Analytic Expressions	57	[53]
	Lagrange's Interpolation Formula	58	[54]
	Taylor's Theorem and Taylor's Series	59	[57]
	Approximation of the Integral and of the Derivative		
	Using Lagrange's Polynomial	63	[59]
	Analytic Functions and Their Use in the Explanation of Nature		[61]
	Interpolation by Means of Trigonometric Series		[67]
			[• ·]
IV.	<u> </u>		
	of Functions	75	[70]
	Evaluation of the Error in the Representation		
	of Empirical Functions	75	[70]
	Trigonometric Interpolation According		
	to the Least Squares Method	78	[72]
	The Harmonic Analyser		[73]
	Examples of Trigonometric Series		[76]
	Chebyshev's Works on Interpolation		[82]
V.	Functions of Two Variables		[84]
	Continuity	91	[84]
	Interchangeability of the Order of Differentiation.		
	Practical Example of a Function for Which $\frac{\partial^2 f}{\partial x \partial y} \neq \frac{\partial^2 f}{\partial y \partial x}$	97	[90]
	Approximate Representation of Functions of the Spherical Surface	21	[JO]
	by Means of Series of Spherical Harmonics	103	[94]
	Distribution of the Values of a Spherical Function	103	[94]
	over the Sphere	110 1	1001
		liu	100]
	Estimate of the Error Truncating the Series of Spherical Harmonics	110 1	1001
	of Spherical Harmonics	112 [102]
Sec	ond Part: Free Geometry of Plane Curves	115 [1041
	one rain from Southern of Finding Out 165 1111111111111111111111111111111111	.10 [נדטב
I.	Precision Geometry Considerations in Plane Geometry 1	17 [104]
	Theorems About Point Sets		
	Point Sets Obtained by Inversion with Respect to Two	•	•
	•	11Q F	1061

Contents xvii

	Properties of These Point Sets
	The Concept of Bi-Dimensional Continuum.
	The General Concept of Curve
	About the Peano-Curve that Fills a Whole Square
	A More Specific Concept of Curve: The Jordan Curve 135 [123]
	Other Limitations to the Concept of Curve: The Regular Curve . 139 [127]
	Approximation of intuitive Curves by Means
	of Regular Functions
	Perception of Idealised Curves
	Classification of Idealised Curves: Analytical and
	Algebraic Curves. Geometrical Construction of the Latter
	as Proposed by Graßmann
	Mastery of the Empirical Phenomena by Means of
	Idealised Structures: Perry's Point of View 146 [134]
II.	Further Considerations of Precision Mathematics on Plane
	Geometry
	Iterated Inversion with Respect to Two Touching Circles 149 [135]
	The Same with Respect to Three Touching Circles
	(Modular Figure)
	The Normal Case of Four Circles Touching Each Other
	in a Cyclic Succession
	The General Case of Four Circles Touching Each Other
	in a Cyclic Succession
	Properties of the Non-Analytic Curves thus Arising 164 [150]
	Premises to the Whole Development.
	Further Idealization by Veronese
III.	Transition to Practical Geometry: a) Geodesy 171 [157]
	Inaccuracy of All Practical Measurements.
	Examples from the Snellius Problem
	Determination of the Precision's Degree
	by Repeated Measurements. Interpretation in Principle
	of the Method of Least Squares
	Approximated Calculations, Explained by Means of
	Legendre's Theorem for Small Spherical Triangles 175 [162]
	The Geodetic Meaning of Shortest Line on the Earth-Spheroid
	(with Postulates Concerning the Theory of
	Differential Equations)
	About the Geoid and its Practical Determination 180 [167]

xviii Contents

IV.	Further Considerations on Practical Geometry: b) Drawing		
	Geometry	185	[171]
	Postulation of a Theory of Errors for Drawing Geometry,		
	Explained Using a Graphic Reproduction		
	of Pascal's Theorem	185	[171]
	About the Possibility to Deduce Properties of the Idealised Curve		
	from the Empirical Shape	189	[176]
	Application of the Procedure on Algebraic Curves in Particular.		
	Prerequisites Assumed in Algebra	191	[178]
	Establishing the Theorem We Want to Prove:		
	$w'+2t''=n(n-2) \ldots \ldots \ldots \ldots \ldots$	196	[183]
	Principles of the Continuity Proof to be Presented	198	[185]
	Transition of a C_n to a Shape with a Double Point	201	[188]
	Examples of Curves for Which the Theorem Holds;		
	Case of Even n		
	The Same for Odd n	209	[195]
	Explanation of the Continuity Proof Referring to Examples.		
	Performing the Proof	212	[198]
Thu	rd Part: About the Perception of Idealised Structures		
	rd Part: About the Perception of Idealised Structures Means of Drawings and Models	219	[205]
by N	Means of Drawings and Models		[205]
by N	Means of Drawings and Models	-	
by N	Means of Drawings and Models	-	
by N	Means of Drawings and Models	-	
by N	Means of Drawings and Models	- 221	[205]
by N	Means of Drawings and Models	- 221 221	[205]
by N	Means of Drawings and Models	- 221 221 230	[205] [205] [213]
by N	About the Perception of Idealised Structures by Means of Drawings and Models Gestalt Relations of Non-Singular Space Curves, in Particular C3 (Projections of the Curve and Plane Sections of its Tangent Surface) The Seven Kinds of Singular Points of Space Curves General Considerations on the Shape of Non-Singular Surfaces	- 221 221 230	[205] [205] [213]
by N	About the Perception of Idealised Structures by Means of Drawings and Models	221 221 230 233	[205] [205] [213] [215]
by N	About the Perception of Idealised Structures by Means of Drawings and Models Gestalt Relations of Non-Singular Space Curves, in Particular C3 (Projections of the Curve and Plane Sections of its Tangent Surface) The Seven Kinds of Singular Points of Space Curves General Considerations on the Shape of Non-Singular Surfaces About the Double Points of the F3, in Particular its Biplanar and Uniplanar Points	221 221 230 233 236	[205] [205] [213] [215] [218]
by N	About the Perception of Idealised Structures by Means of Drawings and Models Gestalt Relations of Non-Singular Space Curves, in Particular C3 (Projections of the Curve and Plane Sections of its Tangent Surface) The Seven Kinds of Singular Points of Space Curves General Considerations on the Shape of Non-Singular Surfaces About the Double Points of the F3, in Particular its Biplanar and Uniplanar Points About the General Behaviour of the F3	221 221 230 233 236	[205] [205] [213] [215] [218]
by N	About the Perception of Idealised Structures by Means of Drawings and Models Gestalt Relations of Non-Singular Space Curves, in Particular C3 (Projections of the Curve and Plane Sections of its Tangent Surface) The Seven Kinds of Singular Points of Space Curves General Considerations on the Shape of Non-Singular Surfaces About the Double Points of the F3, in Particular its Biplanar and Uniplanar Points About the General Behaviour of the F3 Appeal for an Always Renewed Adjustment of the Traditional	221 221 230 233 236	[205] [205] [213] [215] [218]
by N	About the Perception of Idealised Structures by Means of Drawings and Models Gestalt Relations of Non-Singular Space Curves, in Particular C_3 (Projections of the Curve and Plane Sections of its Tangent Surface) The Seven Kinds of Singular Points of Space Curves General Considerations on the Shape of Non-Singular Surfaces About the Double Points of the F_3 , in Particular its Biplanar and Uniplanar Points About the General Behaviour of the F_3 Appeal for an Always Renewed Adjustment of the Traditional Operating Mode of Science by Means of the	221 221 230 233 236 243	[205] [205] [213] [215] [218] [224]
by N	About the Perception of Idealised Structures by Means of Drawings and Models Gestalt Relations of Non-Singular Space Curves, in Particular C3 (Projections of the Curve and Plane Sections of its Tangent Surface) The Seven Kinds of Singular Points of Space Curves General Considerations on the Shape of Non-Singular Surfaces About the Double Points of the F3, in Particular its Biplanar and Uniplanar Points About the General Behaviour of the F3 Appeal for an Always Renewed Adjustment of the Traditional	221 221 230 233 236 243	[205] [205] [213] [215] [218] [224]
by M	About the Perception of Idealised Structures by Means of Drawings and Models Gestalt Relations of Non-Singular Space Curves, in Particular C_3 (Projections of the Curve and Plane Sections of its Tangent Surface) The Seven Kinds of Singular Points of Space Curves General Considerations on the Shape of Non-Singular Surfaces About the Double Points of the F_3 , in Particular its Biplanar and Uniplanar Points About the General Behaviour of the F_3 Appeal for an Always Renewed Adjustment of the Traditional Operating Mode of Science by Means of the	221 221 230 233 236 243	[205] [205] [213] [215] [218] [224]