

Contents

About this book	xi
1 Concept lattices	1
1.1 Examples of concept lattices	1
1.2 Basic notions	3
1.2.1 Formal contexts and cross-tables	4
1.2.2 The derivation operators	8
1.2.3 Formal concepts, extents and intents	10
1.2.4 Conceptual hierarchy	12
1.3 The algebra of concepts	13
1.3.1 Reading a concept lattice diagram	13
1.3.2 Supremum and infimum	16
1.3.3 Complete lattices	17
1.3.4 The basic theorem	18
1.4 How to draw a concept lattice diagram	21
1.4.1 Finding the concepts	21
1.4.2 An example	22
1.4.3 Clarifying and reducing a formal context	27
1.4.4 Computer programs	29
1.5 Further reading	30
1.6 Exercises	31
2 An algorithm for closure systems	39
2.1 Definition and examples	39
2.1.1 Closure systems	40
2.1.2 Closure operators	41
2.1.3 The closure systems of intents and of extents	42
2.1.4 The closure system of closure systems	43
2.2 The NEXT CLOSURE algorithm	44
2.2.1 Representing sets by bit vectors	44
2.2.2 Closures in lexic order	45
2.2.3 Finding formal concepts	48
2.2.4 The complexity of the NEXT CLOSURE algorithm	49

2.3	Computing only certain closed sets	52
2.3.1	Closures that contain and avoid prescribed elements . .	52
2.3.2	Small closures only and large closures only	56
2.3.3	Small generating sets and the TITANIC algorithm	61
2.3.4	Only one of each kind	67
2.4	More about the algorithms	70
2.4.1	Generalizations and proofs	70
2.4.2	The order relation and the lattice diagram	74
2.4.3	Computing closures in the reverse order	77
2.4.4	Other algorithms	79
2.5	Further reading	82
2.6	Exercises	82
3	The canonical basis	87
3.1	Implications	87
3.1.1	Implications of a formal context	88
3.1.2	Implication inference	90
3.1.3	Computing the closure $\mathcal{L}(X)$ of a set X	92
3.1.4	Linear complexity of implication inference	93
3.2	Pseudo-closed sets	94
3.2.1	A recursive definition	95
3.2.2	The canonical basis	96
3.2.3	The size of the canonical basis	97
3.2.4	Making canonical basis implications shorter	99
3.3	Finding pseudo-closed sets	101
3.3.1	Preclosed sets form a closure system	101
3.3.2	An algorithm for computing the canonical basis	103
3.3.3	An example and optimizations	103
3.3.4	Computing the canonical basis from given implications .	105
3.4	Finding important implications only	109
3.4.1	Implications with small premise	109
3.4.2	Implications with high support	112
3.4.3	Association rules	115
3.4.4	Under symmetry	117
3.5	Further reading	121
3.6	Exercises	122
4	Attribute exploration	125
4.1	The exploration algorithm	125
4.1.1	Abstract exploration	126
4.1.2	A concrete algorithm	128
4.1.3	When the exploration terminates	130
4.1.4	Exploration strategy and incompleteness	132

- 4.2 Examples 134
 - 4.2.1 International organizations in Europe 135
 - 4.2.2 Pairs of squares 139
 - 4.2.3 Components of musical experience 145
 - 4.2.4 Lattice-based access control models 147
- 4.3 Variations of the algorithm 156
 - 4.3.1 Object exploration 156
 - 4.3.2 Harmless background knowledge and exceptions 158
 - 4.3.3 Exploration under symmetry 167
 - 4.3.4 Partially given examples 169
- 4.4 A case study in mathematics 174
 - 4.4.1 Simple mathematical structures: bi-unars 175
 - 4.4.2 Generating examples 176
 - 4.4.3 Using background knowledge and symmetries 177
 - 4.4.4 The exploration completed 179
- 4.5 Further reading 184
- 4.6 Exercises 184
- 5 Exploring data tables 187**
 - 5.1 Background clauses and why we need them 188
 - 5.1.1 An unpleasant example 188
 - 5.1.2 Clauses of a formal context 189
 - 5.1.3 Cumulated clauses 191
 - 5.1.4 Contextual attribute logic 192
 - 5.2 A generalized canonical basis 194
 - 5.2.1 Pseudo-models 195
 - 5.2.2 The “canonical basis” of cumulated clauses 197
 - 5.2.3 Finding models of cumulated clauses 200
 - 5.2.4 All models and the basis 202
 - 5.3 Making inference feasible 205
 - 5.3.1 Complexity considerations 206
 - 5.3.2 Implication inference with background knowledge . . . 207
 - 5.3.3 No canonical basis 210
 - 5.3.4 Attribute exploration with background knowledge . . . 212
 - 5.4 Conceptual scaling of data tables 218
 - 5.4.1 A warm-up example 218
 - 5.4.2 Many-valued contexts and their derived contexts 222
 - 5.4.3 Scaling induced background knowledge 230
 - 5.4.4 The implications of a scaled many-valued context 231
 - 5.5 Further reading 234
 - 5.6 Exercises 234

6	More expressive variants of exploration	237
6.1	Rule exploration	237
6.1.1	Triplets of squares	238
6.1.2	Predicates: attributes of power contexts	244
6.1.3	Exploring rules	247
6.1.4	A problem on evolutionary trees	249
6.2	Attribute descriptions	253
6.2.1	Binary power context pairs	254
6.2.2	Terminological attributes	255
6.2.3	Attribute exploration with Description Logics	261
6.2.4	Exploration of terminological attributes	263
6.3	Concept Exploration	265
6.3.1	Sublattice generation via double exploration	266
6.3.2	Two examples of concept exploration	268
6.3.3	Manual concept exploration	275
6.3.4	Distributive concept exploration	280
6.4	Exploration galore!	281
6.4.1	Exploring faulty data	281
6.4.2	Exploration in a fuzzy setting	283
6.4.3	Triadic data	284
6.4.4	Exploration with different queries	287
6.4.5	Other contributions	290
6.5	Further reading	291
	Bibliography	293
	List of Algorithms	311
	Index	312