

Contents

Preface *XV*

Part I Mathematics *1*

1	Functions of One Variable	3
1.1	Limits	3
1.2	Elementary Calculus	5
1.2.1	Differentiation Products and Quotients	6
1.2.2	Chain Rule	7
1.2.3	Inverse Functions	8
1.3	Integration	10
1.4	The Binomial Expansion	14
1.5	Taylor's Series	15
1.6	Extrema	17
1.7	Power Series	17
1.8	Basic Functions	19
1.8.1	Exponential	19
1.8.2	Logarithm	22
1.9	First-Order Ordinary Differential Equations	24
1.10	Trigonometric Functions	25
1.10.1	L'Hôpital's Rule	27
	Problems	27
2	Complex Numbers	29
2.1	Exponential Function of a Complex Variable	30
2.2	Argand Diagrams and the Complex Plane	32
2.3	Complex Logarithm	34
2.4	Hyperbolic Functions	34
2.5	The Simple Harmonic Oscillator	36
2.5.1	Mechanics in One Dimension	38
2.5.2	Damped and Driven Oscillations	40
	Problems	47

3	Vectors in $\mathbb{R}^3$	51
3.1	Basic Operation	51
3.1.1	Scalar Triple Product	55
3.1.2	Vector Equations of Lines and Planes	56
3.2	Kinematics in Three Dimensions	57
3.2.1	Differentiation	57
3.2.2	Motion in a Uniform Magnetic Field	57
3.3	Coordinate Systems	59
3.3.1	Polar Coordinates	59
3.4	Central Forces	60
3.5	Rotating Frames	64
3.5.1	Larmor Effect	66
	Problems	67
4	Vector Spaces	71
4.1	Formal Definition of a Vector Space	71
4.2	Fourier Series	75
4.3	Linear Operators	78
4.4	Change of Basis	89
	Problems	91
5	Functions of Several Variables	95
5.1	Partial Derivatives	95
5.1.1	Definition of the Partial Derivative	95
5.1.2	Total Derivatives	98
5.1.3	Elementary Numerical Methods	104
5.1.4	Change of Variables	107
5.1.5	Mechanics Again	109
5.2	Extrema under Constraint	111
5.3	Multiple Integrals	113
5.3.1	Triple Integrals	116
5.3.2	Change of Variables	117
	Problems	121
6	Vector Fields and Operators	125
6.1	The Gradient Operator	125
6.1.1	Coordinate Systems	127
6.2	Work and Energy in Vectorial Mechanics	130
6.2.1	Line Integrals	133
6.3	A Little Fluid Dynamics	135
6.3.1	Rotational Motion	138
6.3.2	Fields	141
6.4	Surface Integrals	142
6.5	The Divergence Theorem	146
6.6	Stokes' Theorem	149

6.6.1	Conservative Forces	153
	Problems	154
7	Generalized Functions	159
7.1	The Dirac Delta Function	159
7.2	Green's Functions	163
7.3	Delta Function in Three Dimensions	165
	Problems	169
8	Functions of a Complex Variable	173
8.1	Limits	174
8.2	Power Series	178
8.3	Fluids Again	179
8.4	Complex Integration	180
8.4.1	Application of the Residue Theorem	186
	Problems	192
Part II	Physics	195
9	Maxwell's Equations: A Very Short Introduction	197
9.1	Electrostatics: Gauss's Law	197
9.1.1	Conductors	203
9.2	The No Magnetic Monopole Rule	204
9.3	Current	205
9.4	Faraday's Law	206
9.5	Ampère's Law	208
9.6	The Wave Equation	210
9.7	Gauge Conditions	211
	Problems	213
10	Special Relativity: Four-Vector Formalism	217
10.1	Lorentz Transformation	217
10.1.1	Inertial Frames	217
10.1.2	Properties and Consequences of the Lorentz Transformation	220
10.2	Minkowski Space	220
10.2.1	Four Vectors	220
10.2.2	Time Dilation	226
10.3	Four-Velocity	227
10.3.1	Four-Momentum	229
10.4	Electrodynamics	234
10.4.1	Maxwell's Equations in Four-Vector Form	234
10.4.2	Field of a Moving Point Charge	237
10.5	Transformation of the Electromagnetic Fields	239
	Problems	240

11	Quantum Theory	243
11.1	Bohr Atom	243
11.2	The de Broglie Hypothesis	246
11.3	The Schrödinger Wave Equation	246
11.4	Interpretation of the Wave function	249
11.5	Atom	251
11.5.1	The Delta Function Potential	252
11.5.2	Molecules	254
11.6	Formalism	257
11.6.1	Dirac Notation	257
11.7	Probabilistic Interpretation	258
11.7.1	Commutator Relations	259
11.7.2	Functions of Observables	261
11.7.3	Block's Theorem	261
11.7.4	Band Structure	263
11.8	Time Evolution	266
11.9	The Stern–Gerlach Experiment	269
11.9.1	Successive Measurements	270
11.9.2	Spin Space	271
11.9.3	Explicit Matrix Representation	272
11.9.4	Larmor Precession	274
11.9.5	EPR Paradox	275
11.9.6	Bell's Theorem	276
11.9.7	The Harmonic Oscillator	279
	Problems	280
12	An Informal Treatment of Variational Principles and their History	287
12.1	Sin and Death	287
12.2	The Calculus of Variations	288
12.3	Constrained Variations	293
12.4	Hamilton's Equations	293
12.5	Phase Space	296
12.6	Fixed Points	296
	Problems	298
A	Conic Sections	301
A.1	Polar Coordinates	303
A.2	Intersection of a Cone and a Plane	304
B	Vector Relations	305
B.1	Products	305
B.2	Differential Operator Relations	305
B.3	Coordinates	306

Cylindrical Polar 306
Spherical Polar 307

Bibliography 309

Index 311