

Contents

Part I Arts and Nonlinear Systems: 'Nonlinear'

1	Introduction	3
2	Boundaries in Visual Perception and the Arts	9
2.1	Is Our Visual Perception Two Dimensional or Three Dimensional?	11
2.2	Message of the Frame	14
2.3	Importance of the Frame to the Image Inside	17
2.4	What Type of Mathematics Does Our Visual Brain Possess?	25
2.5	Framed Versus Non-Framed	31
2.5.1	The Necessity of a Frame	33
2.5.2	Framed Paintings of Canvases	35
2.5.3	Eliminating Frame Effects	37
2.5.4	Frameless: Greek Pottery, Vermeer, and Feynman	45
2.6	Perception of Image Boundaries	51
2.6.1	Illusions and Frames	53
2.6.2	Biocybernetics	61
2.6.3	Representations of Boundaries in the Left and Right Cerebral Hemispheres	71
2.7	René Magritte and Bernhard Riemann	73
3	Boundaries in Social Systems	79
3.1	Social Science Approach to Boundaries	79
3.1.1	Social and Collective Identity	80
3.1.2	Class, Ethnic, and Gender Inequality	81
3.1.3	Professions, Science, and Knowledge	81
3.1.4	Communities, National Identities, and Spatial Boundaries	82
3.2	Social Boundaries and Networks	82
3.3	Impact of Social Boundaries in Social Relations	84
3.4	Mathematical Approaches to Social Boundaries	87

3.5	Social Distance: Euclidean Metric	89
3.6	Social Distance: Ultrametric	93
3.7	Social Topological Boundaries	95
3.8	Social Topological Patterns	97
3.8.1	Growth Models	97
3.8.2	Cooperation and Patterns	100
3.8.3	Multivariate Networks	101
3.8.4	Pattern Formation in Unstable Social Systems	104

Part II Mathematical Language

4	Continuous Mathematics	111
4.1	Intuitive Introduction to Topology	111
4.1.1	Separation	114
4.1.2	Compactness	117
4.1.3	Connectedness and Connectivity	117
4.2	Topological Boundary	119
4.3	Manifold Boundary	120
4.4	Forms and the Lie Derivative	122
4.5	Fiber Bundles and Covariant Derivative	129
4.6	Is the Lagrangian Derivative a Lie or a Covariant Derivative?	134
4.7	Deformation of the Boundary	140
4.8	Differential Topology of Boundaries: Cobordism	149
5	Discrete Mathematics	155
5.1	Structured Finite Sets	156
5.2	Formal Theory of Graphs	156
5.3	Algebraic Theory and Spectra of Graphs	162
5.3.1	Relations Between Eigenvalues and the Diameter	167
5.3.2	Relations Between Eigenvalues and Connectivity	169
5.3.3	Relations Between Eigenvalues and the Topology of a Graph	169
5.3.4	Relations Between Eigenvalues and Paths	170
5.3.5	Other Relations Between Eigenvalues	171
5.4	Graph Topology and Boundaries	173
5.4.1	The Graph Topology and the Diameter	173
5.4.2	Embeddings	176
5.4.3	Isoperimetric Problems	179
5.4.4	Separations	182
5.4.5	Expanders	184
5.5	Algebraic Topology	186
5.6	Classification of Continuous Structure by Discrete Criteria	193
5.7	Triangulations and CW Complexes	195
5.8	Connecting Discrete and Continuous	198

Part III Applications

6 The Boundary in the Philosophy of Science 203

6.1 Boundaries in Epistemology 204

6.2 Triadic Classifications, Complexity, and Boundaries 205

6.3 Boundarylessness as the Philosophy of Vagueness 207

7 Networks and Their Boundaries 211

7.1 Complex Networks 212

7.2 World Networks 213

7.3 The Shape of the Internet 217

7.4 Internet Is a Boundary..... 226

8 Big Data Systems 229

8.1 Data Dimensionality 230

8.2 Topology of Big Data: Persistent Homology 235

8.3 Topology of Big Data: Regions with Holes 241

9 Physical Boundaries 245

9.1 Geometry of Inviscid Fluids 249

9.2 Geometry of Viscous Fluids 256

9.3 Soap Films with Boundary..... 259

9.4 3D Drops..... 270

9.5 Rotation of 3D Drops 272

9.6 Rotation of 2D Drops 292

9.7 Leidenfrost Drops 294

9.8 Spinning Polygons 300

9.9 Universality in Rotating Fluid Patterns..... 316

9.9.1 Hollow Polygons on a Rotating Fluid Surface..... 316

9.9.2 Polygonal Eyewalls in Hurricanes 322

9.9.3 From the Lab to Saturn 324

9.10 Boundary of Axons and Nerve Pulse Propagation..... 326

10 Conclusions 339

References..... 345

Index..... 357