
Contents

1	Descriptive Synopsis of the Text	1
2	Preliminaries	13
2.1	Causality – Determinism	14
2.2	Dynamical Systems – Examples	21
2.3	Phase Space	28
2.4	First Integrals and Manifolds	30
2.5	Qualitative and Quantitative Approach	34
3	Mathematical Introduction to Dynamical Systems	35
3.1	Linear Autonomous Systems	35
3.2	Non-linear Systems and Stability	47
3.3	Invariant Manifolds	53
3.4	Discretisation in Time	55
3.5	Poincaré Map	57
3.6	Fixed Points and Cycles of Discrete Systems	59
3.7	An Example of Discrete Dynamics – The Logistic Map	63
3.8	Fourier Series and Fourier Transform	70
3.8.1	Fourier Series	70
3.8.2	Fourier Integral and Fourier Transform	73
3.8.3	Characteristics of the Fourier Transform	75
3.8.4	Elementary Fourier Transform, Line Spectra, Dirac δ -Function	79
3.8.5	Wavelet Transform	83
3.9	Basic Concepts of Probability Theory	86
3.9.1	Experiment with Random Outcome	87
3.9.2	Random Variables	90
3.9.3	Probability	90
3.9.4	Conditional Probability and Joint Probability	95
3.9.5	Distribution Function and Probability Density Function ...	97
3.9.6	Characteristic Values of a Distribution	101
3.9.7	Independent and Dependent Events	104
3.9.8	Moment-Generating and Characteristic Functions	106
3.9.9	Special Probability Distributions	109
3.9.10	Central Limit Theorem	116
3.9.11	Cauchy Distribution and α -Stable Lévy Distribution	120

3.9.12	Analysis of Stochastic Processes	126
3.10	Invariant Measure and Ergodic Orbits	130
3.10.1	Natural Invariant Density of the Logistic Map	130
3.10.2	Frobenius-Perron Equation and Ergodicity	135
4	Dynamical Systems without Dissipation	139
4.1	Hamilton Equations for Conservative Systems	139
4.2	Canonical Transformations, Integrability	145
4.3	f-Dimensional Tori and Trajectories	155
4.4	An Outline of the KAM Theory	158
4.5	Unstable Tori, Chaotic Regions	163
4.6	A Numerical Example: The Hénon Map	172
5	Dynamical Systems with Dissipation	189
5.1	Volume Contraction – A Basic Characteristic of Dissipative Systems	190
5.2	Strange Attractor: Lorenz Attractor	192
5.3	Power Spectrum and Autocorrelation	200
5.4	Lyapunov Exponents	205
5.4.1	Linear Stability Analysis of Non-linear Systems: State of Equilibrium	205
5.4.2	Stability of Periodic Solutions: Floquet Theory	211
5.4.3	Lyapunov Exponent of One-Dimensional Maps	221
5.4.4	Lyapunov Exponents of n-Dimensional Continuous Systems	224
5.4.5	Lyapunov Exponents of n-Dimensional Discrete Systems	231
5.4.6	Numerical Calculation of Lyapunov Exponents	232
5.5	Dimensions	238
5.5.1	Cantor Set	240
5.5.2	Fractal Dimensions: Capacity Dimension and Hausdorff-Besicovitch Dimension	244
5.5.3	Information Dimension	246
5.5.4	Correlation Dimension, Pointwise Dimension and Reconstruction of Attractors	258
5.5.5	Generalised Dimension D_q	274
5.5.6	Lyapunov Dimension and Kaplan-Yorke Conjecture	276
5.6	Kohnogorov-Sinai Entropy	281
5.6.1	The Bernoulli Shift	283
5.6.2	Definition of KS Entropy	287
5.6.3	Link between KS Entropy and Lyapunov Exponents	293
5.6.4	Time Span for Reliable Prognoses	296
6	Local Bifurcation Theory	299
6.1	Motivation	301
6.2	Centre Manifold	308
6.3	Normal Forms	328

6.4	Normal Forms of Bifurcations for One-Parametric Flows	340
6.5	Stability of Bifurcations Subject to Perturbations	360
6.6	Bifurcations of the Fixed Points of One-Parametric Maps	364
6.7	Renormalisation and Self-similarity with the Example of the Logistic Map	387
6.7.1	The Mechanism of Period Doublings ad Infinitum	387
6.7.2	Superstable Cycles	393
6.7.3	Self-similarity in the x -Space	398
6.7.4	Self-similarity in the Parameter Space	409
6.7.5	Link with Second-Order Phase Transitions and Renormalisation Methods	422
6.8	A Descriptive Introduction to Synergetics	426
7	Convective Flow: Bénard Problem	435
7.1	Basic Hydrodynamic Equations	441
7.2	Boussinesq-Oberbeck Approximation	451
7.3	Lorenz Model	453
7.4	Evolution of the Lorenz System	459
8	Routes to Chaos	473
8.1	Landau Scenario	473
8.2	Ruelle-Takens Scenario	478
8.2.1	Instability of Quasi-Periodic Motions on the 3D Torus	478
8.2.2	Experiments of Gollub and Swinney	483
8.3	Universal Characteristics of the Transition from Quasi-Periodicity to Chaos	486
8.3.1	The Kicked Rotator with Damping	487
8.3.2	The One-Dimensional Circle Map	490
8.3.3	Scaling Characteristics of the Circle Map	502
8.3.3.1	Local Scaling Laws	502
8.3.3.2	Global Scaling Laws	514
8.4	The Feigenbaum Route to Chaos via Period Doublings	520
8.4.1	Further Scaling Characteristics of the Period Doubling Cascade	523
8.4.2	Experimental Validation of the Feigenbaum Route	534
8.5	Quasi-Periodic Transition for a Fixed Rotation Number	538
8.5.1	Scaling Characteristics of the Quasi-Periodic Transition	539
8.5.2	Multifractal Structures	545
8.5.3	Experimental Validation of the Quasi-Periodic Transition	554
8.6	The Route to Chaos via Intermittency	561
8.6.1	Intermittency in the Logistic Map	562
8.6.2	Classification of Intermittency	567
8.6.3	Type I Intermittency	569
8.6.4	Type III Intermittency	577
8.6.5	Type II Intermittency	582

8.7	Routes Out of Chaos, Control of Chaos	586
8.7.1	Control of Chaos without Feedback	588
8.7.2	Control of Chaos with Feedback	589
9	Turbulence	593
9.1	Dynamics of Incompressible Fluids	597
9.1.1	The Basic Hydrodynamic Equations	598
9.1.2	The Local Energy Dissipation Rate	600
9.1.3	The Vorticity Equation	602
9.1.4	The Lagrangian Description of Fluid Flows	607
9.1.5	Hydrodynamic Vortices	611
9.1.5.1	Lamb-Oseen Vortex	611
9.1.5.2	Stretched Vortex	613
9.1.5.3	Lundgren Vortex	616
9.2	From Chaos to Turbulence	616
9.2.1	Chaos in Fluid Flows	616
9.2.2	Dynamics of Point Vortices in Two-Dimensional Ideal Flows	616
9.2.2.1	The Hamiltonian Character of the Point Vortex Dynamics	618
9.2.2.2	Two Point Vortices	620
9.2.2.3	Three Point Vortices	621
9.2.2.4	Four Point Vortices	626
9.2.2.5	Mixing by Point Vortices	628
9.2.3	Onsager's Equilibrium Theory	628
9.2.3.1	Viscosity	629
9.3	Turbulence: Determinism and Stochasticity	630
9.3.1	Statistical Averaging	630
9.3.2	Moment Equations: The Closure Problem of Turbulence	632
9.3.3	Decaying Turbulence	632
9.3.4	Reynolds Equation and Turbulence Modelling	633
9.4	Characteristic Scales of Turbulence	635
9.4.1	Taylor Hypothesis	635
9.4.2	Phenomenology of Eddy Decay and Energy Cascade	636
9.4.3	The Integral Length L	637
9.4.4	The Kolmogorov Microscales	637
9.4.5	The Taylor Length	638
9.4.6	The Taylor-Reynolds Number	638
9.5	The Turbulent Cascade	639
9.5.1	The von Kármán-Howarth Relation	639
9.5.2	The Energy Spectrum $E(k)$	641
9.5.3	The Energy Cascade in Three-Dimensional Turbulence	643
9.5.4	Heisenberg's Theory	644

9.6	Kolmogorov's Theory of Locally Isotropic Turbulence	646
9.6.1	Evolution Equation of the Velocity Increments	646
9.6.2	The Energy Balance of the Velocity Increment	647
9.6.3	The Averaged Energy Balance Equation	649
9.6.3.1	Homogeneous Turbulence	649
9.6.3.2	Homogeneous Isotropic Turbulence	650
9.6.3.3	Kolmogorov's -4/5 Law	650
9.6.3.4	Dissipation Range	651
9.6.3.5	Inertial Range	652
9.6.4	Kolmogorov's K41 Theory	652
9.6.4.1	Universality of the Statistics of Small-Scale Turbulence	653
9.6.4.2	Self-similar Behaviour in the Inertial Range	653
9.6.4.3	Self-similarity and Fractal Scaling Laws	654
9.6.4.4	Experimental Testing of the Self-similarity Hypothesis	655
9.6.5	The Phenomenon of Intermittency	657
9.6.6	Kolmogorov K62	657
9.6.7	Multifractal Model	660
9.6.8	Multiscaling Statistics	662
9.7	Outlook	668
9.8	Appendix: Isotropic Tensor Fields	669
9.8.1	Longitudinal and Transversal Correlation Functions	671
9.8.1.1	The Correlation Function of Third Order	672
9.8.2	Correlation Functions for Incompressible, Isotropic and Homogeneous Fields	673
9.8.2.1	The Correlation Functions of Second Order	673
9.8.2.2	The Correlation Functions of Third Order	673
9.8.3	Structure Functions of Incompressible Fields	674
9.8.3.1	Structure Function of Second Order	674
9.8.3.2	Structure Function of Third Order	675
10	Computer Experiments	677
10.1	Introduction to Bone Remodelling Processes	680
10.2	Hénon Map	698
10.3	The Lorenz System Revisited	707
10.4	Van der Pol Equation	715
10.4.1	Self-excited Oscillation	715
10.4.2	Driven van der Pol Oscillator	721
10.5	Duffing Equation	737
10.6	Shilnikov Bifurcations	762
10.7	Julia Sets and Their Ordering Principle	771
10.8	Morphology of the Arnold Tongues	788

10.9 Oscillatory Kinetics of Chemical Reactions on Single-Crystal Surfaces	798
10.9.1 Oxidation of Hydrogen on a Platinum Electrode	799
10.9.2 Kinetics of the Catalytic Oxidation of CO on Pt(110)	801
10.9.2.1 Phenomenology of Catalytic Oxidation	802
10.9.2.2 Step by Step Formulation of a Kinetic Model	805
10.9.3 Identification of Chaos and Hyperchaos in Kinetic Surface Reactions	812
10.9.4 Spatio-temporal Pattern Formation	815
References	823
Index	849