

Contents

1	Introduction	1
2	Some Preliminaries	7
2.1	Some Preliminaries in Complex Analysis	7
2.2	The Notion of Capacity	26
2.3	Local Inverse and Analytic Continuation	31
2.4	Uniformly Separated Sequence	33
2.5	Some Results in von Neumann Algebras	40
2.6	Some Results in Operator Theory	46
3	Cowen-Thomson's Theorem	53
3.1	Cowen-Thomson's Theorem on Commutants	53
3.2	Facts from Real and Complex Analysis	58
3.3	Proof of Cowen-Thomson's Theorem	64
3.4	A Proposition on Singularities	76
3.5	An Example not Satisfying Thomson's Condition	78
3.6	Remarks on Chap. 3	84
4	Reducing Subspaces Associated with Finite Blaschke Products	87
4.1	The Distinguished Reducing Subspace	87
4.2	Abelian $\mathcal{V}^*(B)$	95
4.3	Representation for Operators in $\mathcal{V}^*(B)$	100
4.4	Further Consideration on Reducing Subspaces	105
4.5	Proof of Proposition 4.4.6	111
4.6	Abelian $\mathcal{V}^*(B)$ for Order $B = 5, 6$	118
4.7	Remarks on Chap. 4	122
5	Reducing Subspaces Associated with Thin Blaschke Products	125
5.1	Properties of Thin Blaschke Products	125
5.2	Representation for Operators in $\mathcal{V}^*(B)$	129
5.3	Geometric Characterization for $\mathcal{V}^*(B)$	141
5.4	Most M_B Are Irreducible	150
5.5	The Construction of an Example	162

5.6	Another Proof for a Characterization on $\mathcal{V}^*(B)$	170
5.7	Abelian $\mathcal{V}^*(B)$ for Thin Blaschke Products	177
5.8	Finite Blaschke Product Revisited	184
5.9	Remarks on Chap. 5	192
6	Covering Maps and von Neumann Algebras	193
6.1	Regular Branched Covering Maps and Orbifold Domains	193
6.2	Representations of Operators in $\mathcal{V}^*(\phi)$	196
6.3	Abelian $\mathcal{V}^*(\phi)$	207
6.4	Type II Factors Arising from Planar Domains	213
6.5	$\mathcal{V}^*(\phi)$ and Free Group Factors	218
6.6	Type II Factors and Orbifold Domains	226
6.7	Applications to Multi-variable Case	230
6.8	Representation of Operators in $\mathcal{V}_\alpha^*(\phi)$	238
6.9	The Structure of $\mathcal{V}_\alpha^*(\phi)$	242
6.10	Group-Like von Neumann Algebras	245
6.11	Weighted Bergman Spaces over the Upper Half Plane	250
6.12	Remarks on Chap. 6	251
7	Similarity and Unitary Equivalence	253
7.1	The Case of the Hardy Space	253
7.2	Unitary Equivalence on Analytic Multiplication Operators	258
7.3	Similarity of Analytic Toeplitz Operators	260
7.4	Remarks on Chap. 7	268
8	Algebraic Structure and Reducing Subspaces	269
8.1	Algebraic Structure of Essentially Normal Operators	269
8.2	Algebraic Structure and Reducing Subspaces	281
8.3	Monomial Case	283
8.4	More Examples in Multi-variable Case	291
8.5	Remarks on Chap. 8	299
A	Berezin Transform	301
B	Nordgren's Results on Reducing Subspaces	305
C	List of Problems	309
	Bibliography	313
	Index	321