Contents

Preface ---- VII

Part I: Fundamental theory

1	An introduction to higher recursion theory — 3
1.1	Projective predicates —— 3
1.2	Ordinal notations —— 10
1.3	Effective transfinite induction —— 13
1.4	Recursive ordinals —— 16
1.5	Π^1_1 -completeness and Σ^1_1 -boundedness —— 18
2	Hyperarithmetic theory —— 23
2.1	H -sets and Π_2^0 -singletons —— 23
2.2	Δ_1^1 -ness and hyperarithmeticity —— 25
2.3	Spector's Uniqueness Theorem —— 29
2.4	Hyperarithmetic reducibility —— 31
2.5	Some basis theorems and their applications —— 33
2.6	More on ① —— 36
2.7	Codes for sets —— 40
3	Admissibility and constructibility —— 45
3.1	Kripke-Platek set theory —— 45
3.2	Admissible sets —— 51
3.3	Constructibility —— 53
3.4	Projecta and master codes —— 58
3.5	ω -models — 62
3.6	Coding structures —— 64
3.7	The Spector-Gandy Theorem —— 69
4	The theory of Π_1^1 -sets —— 75
4.1	A Π_1^1 -basis theorem —— 75
4.2	Π_1^1 -uniformization —— 77
4.3	Characterizing thin Π_1^1 -sets — 79
4.4	Σ_2^1 -sets — 81
5	Recursion-theoretic forcing —— 85
5.1	Ramified analytical hierarchy —— 85
5.2	Cohen forcing —— 88

5.3	Sacks forcing —— 93
5.4	Characterizing countable admissible ordinals —— 99
6	Set theory —— 105
6.1	Set-theoretic forcing —— 105
6.2	Some examples of forcing —— 109
6.3	A cardinality characterization of Π_1^1 -sets —— 115
6.4	Large cardinals —— 118
6.5	Axiom of determinacy —— 121
6.6	Recursion-theoretic aspects of determinacy —— 123
Part	II: The story of Turing degrees
7	Classification of jump operators 131
7.1	Uniformly degree invariant functions —— 131
7.2	Martin's conjecture for uniformly degree invariant functions — 134
7.3	The Posner–Robinson Theorem —— 137
7.4	Classifying order-preserving functions on 2^{ω} — 141
7.5	Pressdown functions —— 142
8	The construction of Π_1^1 -sets —— 147
8.1	An introduction to inductive definition —— 147
8.2	Inductively defining Π_1^1 -sets of reals —— 150
8.3	$arPi_1^1$ -maximal chains and antichains of Turing degrees —— 154
8.4	Martin's conjecture for Π_1^1 -functions —— 159
9	Independence results in recursion theory —— 165
9.1	Independent sets of Turing degrees —— 165
9.2	Embedding locally finite upper semilattices into $\langle \mathcal{D}, \leq \rangle$ — 170
9.3	Cofinal chains in \mathscr{D} —— 172
9.4	ω -homogeneity of the Turing degrees —— 177
9.5	Some general independence results —— 179
Part	III: Hyperarithmetic degrees and perfect set property
10	Rigidity and biinterpretability of hyperdegrees —— 183
10.1	Embedding lattices into hyperdegrees —— 183
10.2	The rigidity of hyperdegrees —— 186
10.3	Biinterpretability —— 188

11	Basis theorems —— 195
11.1	A basis theorem for Δ_1^1 -sets of reals —— 195
11.2	An antibasis theorem for Π_1^0 -sets —— 199
11.3	Perfect sets in L —— 207
Part IV	/: Higher randomness theory
12	Review of classical algorithmic randomness —— 213
12.1	Randomness via measure theory —— 213
12.2	Randomness via complexity theory —— 216
12.3	Lowness for randomness —— 220
13	More on hyperarithmetic theory —— 223
13.1	Hyperarithmetic measure theory —— 223
13.2	Coding sets above Kleene's ① —— 229
13.3	Hyperarithmetic computation —— 236
14	The theory of higher randomness —— 245
14.1	Higher Kurtz randomness —— 245
14.2	$arDelta_1^1$ and $arPi_1^1$ -Martin-Löf randomness —— 248
14.3	II_1^1 -randomness —— 254
14.4	$\Delta_2^{\hat{1}}$ and $\Sigma_2^{\hat{1}}$ -randomness —— 259
14.5	Kolmogorov complexity and randomness — 262
14.6	Lowness for randomness —— 263
A	Open problems —— 273
A. 1	Hyperarithmetic theory —— 273
A.2	Set-theoretic problems in recursion theory —— 273
A.3	Higher randomness theory —— 274
В	An interview with Gerald E. Sacks —— 275
С	Notations and symbols —— 293
Bibliog	raphy —— 295
Index -	— 303