

Contents

1	Introduction	1
1.1	Geometric Theory of Stochastic Realization	2
1.1.1	Markovian Splitting Subspaces	4
1.1.2	Observability, Constructibility and Minimality	4
1.1.3	Fundamental Representation Theorem	6
1.1.4	Predictor Spaces and Partial Ordering	8
1.1.5	The Frame Space	9
1.1.6	Generalizations	10
1.2	Spectral Factorization and Uniformly Chosen Bases	10
1.2.1	The Linear Matrix Inequality and Hankel Factorization ..	11
1.2.2	Minimality	13
1.2.3	Rational Covariance Extension	13
1.2.4	Uniform Choice of Bases	14
1.2.5	The Matrix Riccati Equation	15
1.3	Applications	16
1.3.1	Smoothing	16
1.3.2	Interpolation	17
1.3.3	Subspace Identification	17
1.3.4	Balanced Model Reduction	19
1.4	An Brief Outline of the Book	21
1.5	Bibliographical Notes	23
2	Geometry of Second-Order Random Processes	25
2.1	Hilbert Space of Second-Order Random Variables	25
2.1.1	Notations and Conventions	26
2.2	Orthogonal Projections	27
2.2.1	Linear Estimation and Orthogonal Projections	28
2.2.2	Facts About Orthogonal Projections	31
2.3	Angles and Singular Values	33
2.3.1	Canonical Correlation Analysis	36
2.4	Conditional Orthogonality	38

2.5	Second-Order Processes and the Shift Operator	40
2.5.1	Stationarity	42
2.6	Conditional Orthogonality and Modeling	44
2.6.1	The Markov Property	44
2.6.2	Stochastic Dynamical Systems.....	47
2.6.3	Factor Analysis	49
2.6.4	Conditional Orthogonality and Covariance Selection	55
2.6.5	Causality and Feedback-Free Processes	57
2.7	Oblique Projections	58
2.7.1	Computing Oblique Projections in the Finite-Dimensional Case	61
2.8	Stationary Increments Processes in Continuous Time	62
2.9	Bibliographical Notes	63
3	Spectral Representation of Stationary Processes	65
3.1	Orthogonal-Increments Processes and the Wiener Integral	65
3.2	Harmonic Analysis of Stationary Processes	70
3.3	The Spectral Representation Theorem	73
3.3.1	Connections to the Classical Definition of Stochastic Fourier Transform	75
3.3.2	Continuous-Time Spectral Representation	77
3.3.3	Remark on Discrete-Time White Noise	78
3.3.4	Real Processes	78
3.4	Vector-Valued Processes	79
3.5	Functionals of White Noise	82
3.5.1	The Fourier Transform	85
3.6	Spectral Representation of Stationary Increment Processes.....	88
3.7	Multiplicity and the Module Structure of $\mathbf{H}(y)$	91
3.7.1	Definition of Multiplicity and the Module Structure of $\mathbf{H}(y)$	92
3.7.2	Bases and Spectral Factorization	95
3.7.3	Processes with an Absolutely Continuous Distribution Matrix	99
3.8	Bibliographical Notes	100
4	Innovations, Wold Decomposition, and Spectral Factorization	103
4.1	The Wiener-Kolmogorov Theory of Filtering and Prediction.....	103
4.1.1	The Role of the Fourier Transform and Spectral Representation	104
4.1.2	Acausal and Causal Wiener Filters	105
4.1.3	Causal Wiener Filtering	108
4.2	Orthonormalizable Processes and Spectral Factorization	110
4.3	Hardy Spaces	115
4.4	Analytic Spectral Factorization	118
4.5	The Wold Decomposition	119
4.5.1	Reversibility	127

4.6	The Outer Spectral Factor.....	129
4.6.1	Invariant Subspaces and the Factorization Theorem	131
4.6.2	Inner Functions	135
4.6.3	Zeros of Outer Functions.....	136
4.7	Toeplitz Matrices and the Szegő Formula.....	138
4.7.1	Algebraic Properties of Toeplitz Matrices.....	146
4.8	Bibliographical Notes	150
5	Spectral Factorization in Continuous Time	153
5.1	The Continuous-Time Wold Decomposition.....	153
5.2	Hardy Spaces of the Half-Plane	154
5.3	Analytic Spectral Factorization in Continuous Time	159
5.3.1	Outer Spectral Factors in $\mathcal{W}^2$	160
5.4	Wide Sense Semimartingales	163
5.4.1	Stationary Increments Semimartingales	166
5.5	Stationary Increments Semimartingales in the Spectral Domain ..	168
5.5.1	Proof of Theorem 5.4.4.....	171
5.5.2	Degenerate Stationary Increments Processes.....	172
5.6	Bibliographical Notes	174
6	Linear Finite-Dimensional Stochastic Systems	175
6.1	Stochastic State Space Models	175
6.2	Anticausal State Space Models	179
6.3	Generating Processes and the Structural Function.....	183
6.4	The Idea of State Space and Coordinate-Free Representation	186
6.5	Observability, Constructibility and Minimality	188
6.6	The Forward and the Backward Predictor Spaces	191
6.7	The Spectral Density and Analytic Spectral Factors	196
6.7.1	The Converse Problem	198
6.8	Regularity	204
6.9	The Riccati Inequality and Kalman Filtering	207
6.10	Bibliographic Notes	213
7	The Geometry of Splitting Subspaces	215
7.1	Deterministic Realization Theory Revisited: The Abstract Idea of State Space Construction	215
7.2	Perpendicular Intersection	217
7.3	Splitting Subspaces.....	220
7.4	Markovian Splitting Subspaces	225
7.5	The Markov Semigroup.....	232
7.6	Minimality and Dimension	234
7.7	Partial Ordering of Minimal Splitting Subspaces.....	238
7.7.1	Uniform Choices of Bases	240
7.7.2	Ordering and Scattering Pairs	243
7.7.3	The Tightest Internal Bounds	246
7.8	Bibliographic Notes	250

8	Markovian Representations	251
8.1	The Fundamental Representation Theorems	252
8.2	Normality, Properness and the Markov Semigroup	257
8.3	The Forward and Backward Systems (The Finite-Dimensional Case)	262
8.4	Reachability, Controllability and the Deterministic Subspace	266
8.5	Markovian Representation of Purely Deterministic Processes	276
8.6	Minimality and Nonminimality of Finite-Dimensional Models ...	281
8.7	Parameterization of Finite-Dimensional Minimal Markovian Representations	284
8.8	Regularity of Markovian Representations	290
8.9	Models Without Observation Noise	294
8.10	The Forward and Backward Systems (The General Case)	297
8.10.1	State-Space Isomorphisms and the Infinite-Dimensional Positive-Real-Lemma Equations ...	306
8.10.2	More About Regularity	309
8.10.3	Models Without Observation Noise	310
8.11	Bibliographical Notes	311
9	Proper Markovian Representations in Hardy Space	313
9.1	Functional Representations of Markovian Representations	313
9.1.1	Spectral Factors and the Structural Function	315
9.1.2	The Inner Triplet of a Markovian Representation	317
9.1.3	State Space Construction	319
9.1.4	The Restricted Shift	323
9.2	Minimality of Markovian Representations	326
9.2.1	Spectral Representation of the Hankel Operators	328
9.2.2	Strictly Noncyclic Processes and Properness	331
9.2.3	The Structural Functions of Minimal Markovian Representations	333
9.2.4	A Geometric Conditions for Minimality	337
9.3	Degeneracy	339
9.3.1	Regularity, Singularity, and Degeneracy of the Error Spaces	340
9.3.2	Degenerate Processes	342
9.3.3	Some Examples	346
9.4	Coercivity Revisited	348
9.5	Models Without Observation Noise	350
9.6	Bibliographical Notes	353
10	Stochastic Realization Theory in Continuous Time	355
10.1	Continuous-Time Stochastic Models	355
10.1.1	Minimality and Nonminimality of Models	356
10.1.2	The Idea of State Space and Markovian Representations	358
10.1.3	Modeling Stationary Processes	360

10.2	Markovian Representations	361
10.2.1	State Space Construction	363
10.2.2	Spectral Factors and the Structural Function	369
10.2.3	From Spectral Factors to Markovian Representations	373
10.3	Forward and Backward Realizations for Finite-Dimensional Markovian Representations.....	376
10.4	Spectral Factorization and Kalman Filtering.....	386
10.4.1	Uniform Choice of Bases	387
10.4.2	Spectral Factorization, the Linear Matrix Inequality and set $\mathcal{P}$	388
10.4.3	The Algebraic Riccati Inequality	393
10.4.4	Kalman Filtering.....	394
10.5	Forward and Backward Stochastic Realizations (The General Case).....	397
10.5.1	Forward State Representation	398
10.5.2	Backward State Representation	403
10.5.3	Stochastic Realizations of a Stationary Process.....	406
10.5.4	Stochastic Realizations of a Stationary-Increment Process	409
10.6	Bibliographical Notes	411
11	Stochastic Balancing and Model Reduction	413
11.1	Canonical Correlation Analysis and Stochastic Balancing.....	414
11.1.1	Observability and Constructibility Gramians	416
11.1.2	Stochastic Balancing	420
11.1.3	Balanced Stochastic Realizations	421
11.2	Stochastically Balanced Realizations from the Hankel Matrix	425
11.3	Basic Principles of Stochastic Model Reduction	430
11.3.1	Stochastic Model Approximation.....	432
11.3.2	Relations to the Maximum Likelihood Criterion.....	437
11.4	Prediction-Error Approximation in Restricted Model Classes	439
11.5	Relative Error Minimization in H^∞	440
11.5.1	A Short Review of Hankel Norm Approximation.....	441
11.5.2	Relative Error Minimization	447
11.6	Stochastically Balanced Truncation	452
11.6.1	The Continuous-Time Case	453
11.6.2	The Discrete-Time Case.....	455
11.6.3	Balanced Discrete-Time Model Reduction.....	459
11.7	Bibliographical Notes	461
12	Finite-Interval and Partial Stochastic Realization Theory	463
12.1	Markovian Representations on a Finite Interval	464
12.2	Kalman Filtering	468
12.2.1	The Invariant Form of the Kalman Filter	470
12.2.2	A Fast Kalman Filtering Algorithm	471
12.3	Realizations of the Finite-Interval Predictor Spaces.....	476

12.4	Partial Realization Theory	480
12.4.1	Partial Realization of Covariance Sequences.....	480
12.4.2	Hankel Factorization of Finite Covariance Sequences....	482
12.4.3	Coherent Bases in the Finite-Interval Predictor Spaces...	486
12.4.4	Finite-Interval Realization by Canonical Correlation Analysis.....	488
12.5	The Rational Covariance Extension Problem	491
12.5.1	The Maximum-Entropy Solution	492
12.5.2	The General Case	496
12.5.3	Determining P from Logarithmic Moments	501
12.6	Bibliographical Notes	505
13	Subspace Identification of Time Series	507
13.1	The Hilbert Space of a Second-Order Stationary Time Series	508
13.2	The Geometric Framework with Finite Data.....	513
13.3	Principles of Subspace Identification.....	515
13.3.1	Coherent Factorizations of Sample Hankel Matrices	516
13.3.2	Approximate Partial Realization	519
13.3.3	Approximate Finite-Interval Stochastic Realization	520
13.3.4	Estimating B and D (The Purely Nondeterministic Case)	522
13.3.5	Estimating B and D (The General Case).....	524
13.3.6	LQ Factorization in Subspace Identification	526
13.4	Consistency of Subspace Identification Algorithms	526
13.4.1	The Data Generating System.....	527
13.4.2	The Main Consistency Result	530
13.4.3	Convergence of the Sample Covariances	531
13.4.4	The Convergence of $(A_N, C_N, \bar{C}_N, \Lambda_N(0))$	535
13.4.5	The Ergodic Case	538
13.4.6	Concluding the Proof of Theorem 13.4.6.....	539
13.4.7	On Order Estimation	540
13.5	Bibliographical Notes	541
14	Zero Dynamics and the Geometry of the Riccati Inequality	543
14.1	The Zero Structure of Minimal Spectral Factors (The Regular Case).....	543
14.1.1	The Discrete-Time Regular Case	548
14.1.2	The Continuous-Time Case	557
14.1.3	Zero Dynamics and Geometric Control Theory.....	565
14.2	Zero Dynamics in the General Discrete-Time Case	566
14.2.1	Output-Induced Subspaces	567
14.2.2	Invariant Directions.....	574
14.3	The Local Frame Space	579
14.3.1	The Geometric Problem	579
14.3.2	The Tightest Local Frame	581

14.4	Invariant Subspaces and the Algebraic Riccati Inequality	583
14.5	Bibliographical Notes	590
15	Smoothing and Interpolation	591
15.1	Smoothing in Discrete Time	592
15.1.1	The Frame Space	594
15.1.2	Two-Filter Formulas	595
15.1.3	Order Reduction in the Nonregular Case	596
15.2	Finite-Interval Realization Theory for Continuous-Time Systems	597
15.2.1	Time-Reversal of the State Equations	598
15.2.2	Forward and Backward Stochastic Realizations	602
15.3	Smoothing in Continuous Time (The General Case)	607
15.3.1	Basic Representation Formulas	607
15.3.2	Mayne-Fraser Two-Filter Formula	610
15.3.3	The Smoothing Formula of Bryson and Frazier	610
15.3.4	The Smoothing Formula of Rauch, Tung and Striebel	611
15.4	Steady-State Smoothers in Continuous-Time	612
15.4.1	The Two-Filter Formula	613
15.4.2	Reduced-Order Smoothing	615
15.5	Steady-State Smoothers in Discrete-Time	620
15.5.1	The Two-Filter Formula	622
15.5.2	Reduced Order Smoothing	624
15.6	Interpolation	628
15.6.1	State Interpolation	628
15.6.2	Output Interpolation	633
15.7	Bibliographical Notes	636
16	Acausal Linear Stochastic Models and Spectral Factorization	637
16.1	Acausal Stochastic Systems	637
16.2	Rational Spectral Factorization	639
16.3	Duality and Rational All-Pass Functions	644
16.3.1	Rational All-Pass Functions	647
16.3.2	Generalizing the Concept of Structural Function	649
16.4	Equivalent Representations of Markovian Splitting Subspaces	652
16.4.1	Invariance with Respect to Duality	652
16.4.2	Invariance with Respect to Pole Structure	653
16.4.3	Invariance with Respect to Zero Structure	656
16.5	The Riccati Inequality and the Algebraic Riccati Equation	657
16.5.1	Zeros of the Spectral Density	659
16.5.2	Zero Flipping by Feedback in Minimal Stochastic Realizations	661
16.5.3	Partial Ordering of the Set $\mathcal{P}$	662
16.5.4	The Solution Set $\mathcal{P}_0$ of the Algebraic Riccati Equation ..	664
16.5.5	Zeros on the Unit Circle Only	667

16.6	Equivalent Representations of Stochastic Realizations, Continued	667
16.6.1	The Structure of Rational All-Pass Functions	670
16.7	Bibliographic Notes	672
17	Stochastic Systems with Inputs	675
17.1	Causality and Feedback	676
17.2	Oblique Markovian Splitting Subspaces	680
17.2.1	Coordinate-Free Representation of Stochastic Systems with Inputs	680
17.3	State Space Construction from Basic Geometric Principles	684
17.3.1	One-Step-Ahead Oblique Markovian Splitting Subspaces	688
17.3.2	The Oblique Predictor Space	691
17.4	Geometric Theory in the Absence of Feedback	695
17.4.1	Feedback-Free Oblique Splitting Subspaces	697
17.4.2	Observability, Constructibility and Minimality	698
17.4.3	The Feedback-Free Oblique Predictor Space	702
17.4.4	Extended Scattering Pairs	703
17.4.5	Stochastic and Deterministic Minimality	707
17.5	Applications to Subspace Identification	710
17.5.1	The Basic Idea of Subspace Identification	711
17.5.2	Finite-Interval Identification	713
17.5.3	The N4SID Algorithm	717
17.5.4	Conditioning in Subspace Identification	721
17.5.5	Subspace Identification with Feedback	721
17.6	Bibliographical Notes	724
A	Basic Principles of Deterministic Realization Theory	725
A.1	Realization Theory	725
A.1.1	The Hankel Factorization	727
A.1.2	Solving the Realization Problem	728
A.2	Balancing	733
A.3	Bibliographical Notes	735
B	Some Topics in Linear Algebra and Hilbert Space Theory	737
B.1	Some Facts from Linear Algebra and Matrix Theory	737
B.1.1	Inner Product Spaces and Matrix Norms	737
B.1.2	Cholesky Factorization	740
B.1.3	Sylvester's Inequality	741
B.1.4	The Moore-Penrose Pseudo-inverse	741
B.1.5	Connections to Least-Squares Problems	744
B.1.6	Matrix Inversion Lemma	746
B.1.7	Logarithm of a Matrix	747
B.1.8	Lyapunov Equations	747
B.1.9	Inertia Theorems	749

B.2	Hilbert Spaces	750
B.2.1	Operators and Their Adjoints	753
B.3	Subspace Algebra	756
B.3.1	The Shift Acting on Subspaces	758
B.4	Bibliographical Notes	759
Bibliography		761
Index		775