

Contents

Preface — v

1 Euclidean Space — 1

- 1.1 Vector spaces — 1
- 1.2 Linear transformations — 6
- 1.3 Determinants — 12
- 1.4 Euclidean spaces — 19
- 1.5 Subspaces of Euclidean space — 25
- 1.6 Determinants as volume — 27
- 1.7 Elementary topology of Euclidean spaces — 30
- 1.8 Sequences — 36
- 1.9 Limits and continuity — 41
- 1.10 Exercises — 48

2 Differentiation — 57

- 2.1 The derivative — 57
- 2.2 Basic properties of the derivative — 62
- 2.3 Differentiation of integrals — 67
- 2.4 Curves — 69
- 2.5 The inverse and implicit function theorems — 75
- 2.6 The spectral theorem and scalar products — 81
- 2.7 Taylor polynomials and extreme values — 89
- 2.8 Vector fields — 94
- 2.9 Lie brackets — 103
- 2.10 Partitions of unity — 108
- 2.11 Exercises — 110

3 Manifolds — 117

- 3.1 Submanifolds of Euclidean space — 117
- 3.2 Differentiable maps on manifolds — 124
- 3.3 Vector fields on manifolds — 129
- 3.4 Lie groups — 137
- 3.5 The tangent bundle — 141
- 3.6 Covariant differentiation — 143
- 3.7 Geodesics — 148
- 3.8 The second fundamental tensor — 153
- 3.9 Curvature — 156
- 3.10 Sectional curvature — 160
- 3.11 Isometries — 163
- 3.12 Exercises — 168

4	Integration on Euclidean space — 177
4.1	The integral of a function over a box — 177
4.2	Integrability and discontinuities — 181
4.3	Fubini's theorem — 187
4.4	Sard's theorem — 195
4.5	The change of variables theorem — 198
4.6	Cylindrical and spherical coordinates — 202
4.6.1	Cylindrical coordinates — 202
4.6.2	Spherical coordinates — 206
4.7	Some applications — 210
4.7.1	Mass — 211
4.7.2	Center of mass — 211
4.7.3	Moment of inertia — 213
4.8	Exercises — 214
5	Differential Forms — 221
5.1	Tensors and tensor fields — 221
5.2	Alternating tensors and forms — 224
5.3	Differential forms — 232
5.4	Integration on manifolds — 236
5.5	Manifolds with boundary — 240
5.6	Stokes' theorem — 243
5.7	Classical versions of Stokes' theorem — 246
5.7.1	An application: the polar planimeter — 249
5.8	Closed forms and exact forms — 252
5.9	Exercises — 257
6	Manifolds as metric spaces — 267
6.1	Extremal properties of geodesics — 267
6.2	Jacobi fields — 271
6.3	The length function of a variation — 275
6.4	The index form of a geodesic — 278
6.5	The distance function — 283
6.6	The Hopf-Rinow theorem — 285
6.7	Curvature comparison — 289
6.8	Exercises — 292
7	Hypersurfaces — 301
7.1	Hypersurfaces and orientation — 301
7.2	The Gauss map — 304
7.3	Curvature of hypersurfaces — 308
7.4	The fundamental theorem for hypersurfaces — 313

7.5	Curvature in local coordinates —	316
7.6	Convexity and curvature —	318
7.7	Ruled surfaces —	320
7.8	Surfaces of revolution —	323
7.9	Exercises —	328

Appendix A —	339
--------------	-----

Appendix B —	345
--------------	-----

Index —	351
---------	-----