

Contents

1. Elementary Newtonian Mechanics	1
1.1 Newton's Laws (1687) and Their Interpretation	1
1.2 Uniform Rectilinear Motion and Inertial Systems	4
1.3 Inertial Frames in Relative Motion	6
1.4 Momentum and Force	6
1.5 Typical Forces. A Remark About Units	8
1.6 Space, Time, and Forces	10
1.7 The Two-Body System with Internal Forces	11
1.7.1 Center-of-Mass and Relative Motion	11
1.7.2 Example: The Gravitational Force Between Two Celestial Bodies (Kepler's Problem)	13
1.7.3 Center-of-Mass and Relative Momentum in the Two-Body System	19
1.8 Systems of Finitely Many Particles	20
1.9 The Principle of Center-of-Mass Motion	21
1.10 The Principle of Angular-Momentum Conservation	21
1.11 The Principle of Energy Conservation	22
1.12 The Closed n -Particle System	23
1.13 Galilei Transformations	24
1.14 Space and Time with Galilei Invariance	27
1.15 Conservative Force Fields	29
1.16 One-Dimensional Motion of a Point Particle	32
1.17 Examples of Motion in One Dimension	34
1.17.1 The Harmonic Oscillator	34
1.17.2 The Planar Mathematical Pendulum	36
1.18 Phase Space for the n -Particle System (in \mathbb{R}^3)	37
1.19 Existence and Uniqueness of the Solutions of $\dot{\mathbf{x}} = \mathcal{F}(\mathbf{x}, t)$	38
1.20 Physical Consequences of the Existence and Uniqueness Theorem	40
1.21 Linear Systems	42
1.21.1 Linear, Homogeneous Systems	42
1.21.2 Linear, Inhomogeneous Systems	43
1.22 Integrating One-Dimensional Equations of Motion	43
1.23 Example: The Planar Pendulum for Arbitrary Deviations from the Vertical	45

1.24 Example: The Two-Body System with a Central Force	48
1.25 Rotating Reference Systems: Coriolis and Centrifugal Forces	55
1.26 Examples of Rotating Reference Systems	56
1.27 Scattering of Two Particles that Interact via a Central Force: Kinematics	64
1.28 Two-Particle Scattering with a Central Force: Dynamics	68
1.29 Example: Coulomb Scattering of Two Particles with Equal Mass and Charge	72
1.30 Mechanical Bodies of Finite Extension	76
1.31 Time Averages and the Virial Theorem	80
Appendix: Practical Examples	82
 2. The Principles of Canonical Mechanics	89
2.1 Constraints and Generalized Coordinates	89
2.1.1 Definition of Constraints	89
2.1.2 Generalized Coordinates	91
2.2 D'Alembert's Principle	91
2.2.1 Definition of Virtual Displacements	91
2.2.2 The Static Case	92
2.2.3 The Dynamical Case	92
2.3 Lagrange's Equations	94
2.4 Examples of the Use of Lagrange's Equations	95
2.5 A Digression on Variational Principles	97
2.6 Hamilton's Variational Principle (1834)	100
2.7 The Euler–Lagrange Equations	100
2.8 Further Examples of the Use of Lagrange's Equations	101
2.9 A Remark About Nonuniqueness of the Lagrangian Function	103
2.10 Gauge Transformations of the Lagrangian Function	104
2.11 Admissible Transformations of the Generalized Coordinates	105
2.12 The Hamiltonian Function and Its Relation to the Lagrangian Function L	106
2.13 The Legendre Transformation for the Case of One Variable	107
2.14 The Legendre Transformation for the Case of Several Variables	109
2.15 Canonical Systems	110
2.16 Examples of Canonical Systems	111
2.17 The Variational Principle Applied to the Hamiltonian Function	113
2.18 Symmetries and Conservation Laws	114
2.19 Noether's Theorem	115
2.20 The Generator for Infinitesimal Rotations About an Axis	117
2.21 More About the Rotation Group	119
2.22 Infinitesimal Rotations and Their Generators	121
2.23 Canonical Transformations	123
2.24 Examples of Canonical Transformations	127
2.25 The Structure of the Canonical Equations	128
2.26 Example: Linear Autonomous Systems in One Dimension	129

2.27	Canonical Transformations in Compact Notation	131
2.28	On the Symplectic Structure of Phase Space	133
2.29	Liouville's Theorem	136
2.29.1	The Local Form	137
2.29.2	The Global Form	138
2.30	Examples for the Use of Liouville's Theorem	139
2.31	Poisson Brackets	142
2.32	Properties of Poisson Brackets	145
2.33	Infinitesimal Canonical Transformations	147
2.34	Integrals of the Motion	148
2.35	The Hamilton–Jacobi Differential Equation	151
2.36	Examples for the Use of the Hamilton–Jacobi Equation	152
2.37	The Hamilton–Jacobi Equation and Integrable Systems	156
2.37.1	Local Rectification of Hamiltonian Systems	156
2.37.2	Integrable Systems	160
2.37.3	Angle and Action Variables	165
2.38	Perturbing Quasiperiodic Hamiltonian Systems	166
2.39	Autonomous, Nondegenerate Hamiltonian Systems in the Neighborhood of Integrable Systems	169
2.40	Examples. The Averaging Principle	170
2.40.1	The Anharmonic Oscillator	170
2.40.2	Averaging of Perturbations	172
2.41	Generalized Theorem of Noether	174
	Appendix: Practical Examples	182
3.	The Mechanics of Rigid Bodies	187
3.1	Definition of Rigid Body	187
3.2	Infinitesimal Displacement of a Rigid Body	189
3.3	Kinetic Energy and the Inertia Tensor	191
3.4	Properties of the Inertia Tensor	193
3.5	Steiner's Theorem	197
3.6	Examples of the Use of Steiner's Theorem	198
3.7	Angular Momentum of a Rigid Body	203
3.8	Force-Free Motion of Rigid Bodies	205
3.9	Another Parametrization of Rotations: The Euler Angles	207
3.10	Definition of Eulerian Angles	209
3.11	Equations of Motion of Rigid Bodies	210
3.12	Euler's Equations of Motion	213
3.13	Euler's Equations Applied to a Force-Free Top	216
3.14	The Motion of a Free Top and Geometric Constructions	220
3.15	The Rigid Body in the Framework of Canonical Mechanics	223
3.16	Example: The Symmetric Children's Top in a Gravitational Field	227
3.17	More About the Spinning Top	229
3.18	Spherical Top with Friction: The "Tippe Top"	231

3.18.1 Conservation Law and Energy Considerations	232
3.18.2 Equations of Motion and Solutions with Constant Energy	234
Appendix: Practical Examples	238
4. Relativistic Mechanics	241
4.1 Failures of Nonrelativistic Mechanics	242
4.2 Constancy of the Speed of Light	245
4.3 The Lorentz Transformations	246
4.4 Analysis of Lorentz and Poincaré Transformations	252
4.4.1 Rotations and Special Lorentz Transformations ("Boosts")	254
4.4.2 Interpretation of Special Lorentz Transformations	258
4.5 Decomposition of Lorentz Transformations into Their Components	259
4.5.1 Proposition on Orthochronous, Proper Lorentz Transformations	259
4.5.2 Corollary of the Decomposition Theorem and Some Consequences	261
4.6 Addition of Relativistic Velocities	264
4.7 Galilean and Lorentzian Space–Time Manifolds	266
4.8 Orbital Curves and Proper Time	270
4.9 Relativistic Dynamics	272
4.9.1 Newton's Equation	272
4.9.2 The Energy–Momentum Vector	274
4.9.3 The Lorentz Force	277
4.10 Time Dilatation and Scale Contraction	279
4.11 More About the Motion of Free Particles	281
4.12 The Conformal Group	284
5. Geometric Aspects of Mechanics	285
5.1 Manifolds of Generalized Coordinates	286
5.2 Differentiable Manifolds	289
5.2.1 The Euclidean Space \mathbb{R}^n	289
5.2.2 Smooth or Differentiable Manifolds	291
5.2.3 Examples of Smooth Manifolds	293
5.3 Geometrical Objects on Manifolds	297
5.3.1 Functions and Curves on Manifolds	298
5.3.2 Tangent Vectors on a Smooth Manifold	300
5.3.3 The Tangent Bundle of a Manifold	302
5.3.4 Vector Fields on Smooth Manifolds	303
5.3.5 Exterior Forms	307
5.4 Calculus on Manifolds	309
5.4.1 Differentiable Mappings of Manifolds	309
5.4.2 Integral Curves of Vector Fields	311

5.4.3	Exterior Product of One-Forms	313
5.4.4	The Exterior Derivative	315
5.4.5	Exterior Derivative and Vectors in \mathbb{R}^3	317
5.5	Hamilton–Jacobi and Lagrangian Mechanics	319
5.5.1	Coordinate Manifold Q , Velocity Space TQ , and Phase Space T^*Q	319
5.5.2	The Canonical One-Form on Phase Space	323
5.5.3	The Canonical, Symplectic Two-Form on M	326
5.5.4	Symplectic Two-Form and Darboux’s Theorem	328
5.5.5	The Canonical Equations	331
5.5.6	The Poisson Bracket	334
5.5.7	Time-Dependent Hamiltonian Systems	337
5.6	Lagrangian Mechanics and Lagrange Equations	339
5.6.1	The Relation Between the Two Formulations of Mechanics	339
5.6.2	The Lagrangian Two-Form	341
5.6.3	Energy Function on TQ and Lagrangian Vector Field	342
5.6.4	Vector Fields on Velocity Space TQ and Lagrange Equations	344
5.6.5	The Legendre Transformation and the Correspondence of Lagrangian and Hamiltonian Functions	346
5.7	Riemannian Manifolds in Mechanics	349
5.7.1	Affine Connection and Parallel Transport	350
5.7.2	Parallel Vector Fields and Geodesics	352
5.7.3	Geodesics as Solutions of Euler–Lagrange Equations	353
5.7.4	Example: Force-Free Asymmetric Top	354
6.	Stability and Chaos	357
6.1	Qualitative Dynamics	357
6.2	Vector Fields as Dynamical Systems	358
6.2.1	Some Definitions of Vector Fields and Their Integral Curves	360
6.2.2	Equilibrium Positions and Linearization of Vector Fields	362
6.2.3	Stability of Equilibrium Positions	365
6.2.4	Critical Points of Hamiltonian Vector Fields	369
6.2.5	Stability and Instability of the Free Top	371
6.3	Long-Term Behavior of Dynamical Flows and Dependence on External Parameters	373
6.3.1	Flows in Phase Space	374
6.3.2	More General Criteria for Stability	375
6.3.3	Attractors	378
6.3.4	The Poincaré Mapping	382
6.3.5	Bifurcations of Flows at Critical Points	386
6.3.6	Bifurcations of Periodic Orbits	390

6.4	Deterministic Chaos	392
6.4.1	Iterative Mappings in One Dimension	392
6.4.2	Qualitative Definitions of Deterministic Chaos	394
6.4.3	An Example: The Logistic Equation	398
6.5	Quantitative Measures of Deterministic Chaos	403
6.5.1	Routes to Chaos	403
6.5.2	Liapunov Characteristic Exponents	407
6.5.3	Strange Attractors	409
6.6	Chaotic Motions in Celestial Mechanics	411
6.6.1	Rotational Dynamics of Planetary Satellites	411
6.6.2	Orbital Dynamics of Asteroids with Chaotic Behavior	417
7.	Continuous Systems	421
7.1	Discrete and Continuous Systems	421
7.2	Transition to the Continuous System	425
7.3	Hamilton's Variational Principle for Continuous Systems	427
7.4	Canonically Conjugate Momentum and Hamiltonian Density	429
7.5	Example: The Pendulum Chain	430
7.6	Comments and Outlook	434
	Exercises	439
	Chapter 1: Elementary Newtonian Mechanics	439
	Chapter 2: The Principles of Canonical Mechanics	446
	Chapter 3: The Mechanics of Rigid Bodies	454
	Chapter 4: Relativistic Mechanics	457
	Chapter 5: Geometric Aspects of Mechanics	460
	Chapter 6: Stability and Chaos	463
	Solution of Exercises	467
	Chapter 1: Elementary Newtonian Mechanics	467
	Chapter 2: The Principles of Canonical Mechanics	483
	Chapter 3: The Mechanics of Rigid Bodies	503
	Chapter 4: Relativistic Mechanics	511
	Chapter 5: Geometric Aspects of Mechanics	523
	Chapter 6: Stability and Chaos	528
	Appendix	537
	A. Some Mathematical Notions	537
	B. Historical Notes	540
	Bibliography	547
	Index	549