## **Contents**

## Preface ---- vii

1	Introduction —— 1
1.1	Why study numerical methods? —— 1
1.2	Terminology —— 2
1.3	Convergence terminology —— 4
1.4	Exercises —— 5
2	Computer representation of numbers and roundoff error —— 7
2.1	Examples of the effects of roundoff error — 7
2.2	Binary numbers —— 10
2.3	64 bit floating point numbers —— 12
2.3.1	Avoid adding large and small numbers —— 14
2.3.2	Subtracting two nearly equal numbers is bad —— 14
2.4	Exercises —— 16
3	Solving linear systems of equations —— 17
3.1	Linear systems of equations and solvability —— 17
3.2	Solving triangular systems —— 19
3.3	Gaussian elimination —— 21
3.4	The backslash operator —— 25
3.5	LU decomposition —— 25
3.6	Exercises —— 26
4	Finite difference methods —— 28
4.1	Approximating the first derivative —— 29
4.1.1	Forward and backward differences —— 29
4.1.2	Centered difference —— 32
4.1.3	Three point difference formulas —— 35
4.1.4	Further notes —— 36
4.2	Approximating the second derivative —— 36
4.3	Application: Initial value ODE's using the forward Euler method —— 37
4.4	Application: Boundary value ODE's —— 40
4.5	Exercises —— 45
5	Solving nonlinear equations —— 46
5.1	The bisection method —— 47
5.2	Newton's method —— 51
5.3	Secant method —— 54

5.4	Comparing bisection, Newton, secant method —— 54
5.5	Combining secant and bisection and the fzero command —— 55
5.6	Equation solving in higher dimensions —— 57
5.7	Exercises —— 59
6	Accuracy in solving linear systems —— 61
6.1	Gauss-Jordan elimination and finding matrix inverses —— 61
6.2	Matrix and vector norms and condition number —— 64
6.3	Sensitivity in linear system solving —— 66
6.4	Exercises —— 68
7	Eigenvalues and eigenvectors —— 69
7.1	Mathematical definition —— 69
7.2	Power method —— 71
7.3	Application: Population dynamics —— 74
7.4	Exercises —— 75
В	Fitting curves to data —— 77
8.1	Interpolation —— 77
8.1.1	Interpolation by a single polynomial —— 77
8.1.2	Piecewise polynomial interpolation —— 80
8.2	Curve fitting —— 83
8.2.1	Line of best fit —— 83
8.2.2	Curve of best fit —— 86
8.3	Exercises —— 89
9	Numerical integration —— 91
9.1	Newton-Cotes methods —— 91
9.2	Composite rules —— 95
9.3	MATLAB's integral function —— 99
9.4	Gauss quadrature —— 99
9.5	Exercises —— 102
10	Initial value ODEs —— 104
10.1	Reduction of higher order ODEs to first order —— 104
10.2	Common methods and derivation from integration rules —— 106
10.2.1	Backward Euler —— 107
10.2.2	Crank–Nicolson —— 108
10.2.3	Runge-Kutta 4 —— 108
10.3	Comparison of speed of implicit versus explicit solvers —— 109
10.4	Stability of ODE solvers —— 111
10.4.1	Stability of forward Euler —— 111

10.4.2	Stability of backward Euler —— 112
10.4.3	Stability of Crank–Nicolson —— 114
10.4.4	Stability of Runge-Kutta 4 —— 115
10.5	Accuracy of ODE solvers —— 115
10.5.1	Forward Euler —— 115
10.5.2	Backward Euler —— 116
10.5.3	Crank–Nicolson —— 117
10.5.4	Runge-Kutta 4 —— 118
10.6	Summary, general strategy, and MATLAB ODE solvers —— 119
10.7	Exercises —— 121
A	Getting started with Octave and MATLAB —— 123
A.1	Basic operations —— 123
A.2	Arrays 126
A.3	Operating on arrays —— 129
A.4	Script files —— 131
A.5	Function files —— 132
A.5.1	Inline functions —— 132
A.5.2	Passing functions to other functions —— 133
A.6	Outputting information —— 133
A.7	Programming in MATLAB —— 134
A.8	Plotting —— 135

Exercises — 136

A.9