## **Contents**

1	Intr	oduction	1			
2	Genetic Background					
	2.1	Basic Genetic Forces: Drift, Mutation, Recombination				
		and Selection	3			
	2.2	Cannings, Wright-Fisher and Moran Models				
		and the Coalescent	6			
		2.2.1 Cannings and Wright-Fisher Models	6			
		2.2.2 Kingman-Tajima Coalescent	7			
	2.3	The Master Equation and the Moran Model	10			
		2.3.1 Discrete Case	10			
		2.3.2 Continuous Case	11			
		2.3.3 Connection with the Master Equation	12			
	2.4	Derivation of the Master Equation	12			
	2.5	Examples of Markov Mutations	14			
		2.5.1 Microsatellite DNA and the Stepwise Mutation Model	14			
		2.5.2 Mitochondrial DNA and the Infinitely Many				
		Sites Model	15			
	Refe	erences	16			
3	Motivating Example: Population Bottlenecks in the History					
	of Modern Humans, Use of the Imbalance Index					
		erences	22			
4	Mathematical Tools					
	4.1	Banach Spaces $l^1$ and $\mathcal{M} = l^1 \otimes l^1 \dots \dots$	23			
		4.1.1 Linear Space	24			
		4.1.2 Normed Space	25			
		4.1.3 Completeness	26			

vi Contents

		4.1.4	Banach Space with Schauder Basis	27			
		4.1.5	Riemann Integral in a Banach Space	28			
		4.1.6	$\mathcal{M}$ as a Tensor Product of Two Copies of $l^1 \dots \dots$	29			
	4.2	Operat	tors and Families of Operators	31			
		4.2.1	Operators in $l^1$ Related to Stochastic Dynamics	31			
		4.2.2	The Space of Bounded Linear Operators	32			
		4.2.3	Convergence of Operators	35			
	4.3	Marko	ov Chains and Semigroups of Operators in $l^1$	37			
		4.3.1	Markov Chains and Their Semigroups	37			
		4.3.2	Finite-State Markov Chains and Kolmogorov Matrices	40			
		4.3.3	Generators of Markov Chains' Semigroups	43			
		4.3.4	From Kolmogorov Matrices to Semigroups	47			
		4.3.5	Generators and Resolvents	49			
		4.3.6	The Semigroups Related to a Kolmogorov Matrix	55			
		4.3.7	Cores of Generators	60			
		4.3.8	Tensor Product Semigroups	62			
	Refe	erences		64			
5	Master Equation and Asymptotic Behavior of Its Solutions 6						
	5.1	_	mics of Joint Distributions	67			
	5.2	The Related Evolution Family					
	5.3		Differential Equation for S	71			
	5.4						
	5.5	Asymptotically Constant Populations					
	5.6	•	ations Growing to Infinity	76			
	5.7		ples of Application of Asymptotics for Master Equation	79			
			Microsatellite Evolution with Allele Size Constraints	79			
			Durrett-Kruglyak Model of Microsatellite Evolution	80			
	5.8		mmary and Notes	81			
				82			
				- <b>-</b>			
6	Epi	logue .		85			
T,	dov			87			
411	uta.			U /			