

Contents

1	Introduction	1
1.1	Background	1
1.2	Synopsis	3
	References	5
2	Mathematical Preliminaries	7
2.1	Lattice Theory	7
2.2	Induction and Coinduction	10
2.3	Topological Spaces	13
2.4	Metric Spaces	15
2.5	Probability Spaces	18
2.6	Linear Programming	19
	References	22
3	Probabilistic Bisimulation	23
3.1	Introduction	23
3.2	Probabilistic Labelled Transition Systems	25
3.3	Lifting Relations	27
3.4	Justifying the Lifting Operation	32
3.4.1	Justification by the Kantorovich Metric	32
3.4.2	Justification by Network Flow	38
3.5	Probabilistic Bisimulation	42
3.6	Logical Characterisation	44
3.6.1	An Adequate Logic	45
3.6.2	An Expressive Logic	48
3.7	Metric Characterisation	52
3.8	Algorithmic Characterisation	56
3.8.1	A Partition Refinement Algorithm	56
3.8.2	An “On-the-Fly” Algorithm	59
3.9	Bibliographic Notes	63

3.9.1	Probabilistic Models	63
3.9.2	Probabilistic Bisimulation	64
	References	65
4	Probabilistic Testing Semantics	71
4.1	A General Testing Framework	71
4.2	Testing Probabilistic Processes	73
4.3	Bounded Continuity	77
4.4	Reward Testing	81
4.4.1	A Geometric Property	81
4.4.2	Nonnegative Rewards	84
4.5	Extremal Reward Testing	85
4.6	Extremal Reward Testing Versus Resolution-Based Reward Testing	88
4.6.1	Must Testing	89
4.6.2	May Testing	92
4.7	Vector-Based Testing Versus Scalar Testing	97
4.8	Bibliographic Notes	100
	References	101
5	Testing Finite Probabilistic Processes	103
5.1	Introduction	103
5.2	The Language pCSP	105
5.2.1	The Syntax	105
5.2.2	The Operational Semantics	106
5.2.3	The Precedence of Probabilistic Choice	108
5.2.4	Graphical representation of pCSP processes	108
5.2.5	Testing pCSP Processes	109
5.3	Counterexamples	112
5.4	Must Versus May Testing	116
5.5	Forward and Failure Simulation	118
5.5.1	The Simulation Preorders	120
5.5.2	The Simulation Preorders are Precongruences	125
5.5.3	Simulations Are Sound for Testing Preorders	127
5.6	A Modal Logic	133
5.7	Characteristic Tests	135
5.8	Equational Theories	137
5.9	Inequational Theories	138
5.10	Completeness	140
5.11	Bibliographic Notes	143
5.11.1	Probabilistic Equivalences	144
5.11.2	Probabilistic Simulations	145
	References	146

6	Testing Finitary Probabilistic Processes	149
6.1	Introduction	149
6.2	The Language rpCSP	152
6.3	A General Definition of Weak Derivations	153
6.3.1	Lifting Relations	154
6.3.2	Weak Transitions	159
6.3.3	Properties of Weak Transitions	162
6.4	Testing rpCSP Processes	167
6.4.1	Testing with Extremal Derivatives	168
6.4.2	Comparison with Resolution-Based Testing	172
6.5	Generating Weak Derivatives in a Finitary pLTS	175
6.5.1	Finite Generability	176
6.5.2	Realising Payoffs	181
6.5.3	Consequences	186
6.6	The Failure Simulation Preorder	191
6.6.1	Two Equivalent Definitions and Their Rationale	192
6.6.2	A Simple Failure Similarity for Finitary Processes	196
6.6.3	Precongruence	198
6.6.4	Soundness	205
6.7	Failure Simulation is Complete for Must Testing	206
6.7.1	Inductive Characterisation	207
6.7.2	The Modal Logic	211
6.7.3	Characteristic Tests for Formulae	213
6.8	Simulations and May Testing	219
6.8.1	Soundness	220
6.8.2	Completeness	222
6.9	Real-Reward Testing	223
6.10	Summary	229
	References	230
7	Weak Probabilistic Bisimulation	233
7.1	Introduction	233
7.2	A Simple Bisimulation	234
7.3	Compositionality	238
7.4	Reduction Barbed Congruence	240
7.5	Bibliographic Notes	244
	References	244
	Index	247