

# Contents

<b>1</b>	<b>Fundamental Equations of Laminated Beams, Plates and Shells.</b>	<b>1</b>
1.1	Three-Dimensional Elasticity Theory in Curvilinear Coordinates	1
1.2	Fundamental Equations of Thin Laminated Shells	3
1.2.1	Kinematic Relations	3
1.2.2	Stress-Strain Relations and Stress Resultants	5
1.2.3	Energy Functions	9
1.2.4	Governing Equations and Boundary Conditions	11
1.3	Fundamental Equations of Thick Laminated Shells	16
1.3.1	Kinematic Relations	17
1.3.2	Stress-Strain Relations and Stress Resultants	18
1.3.3	Energy Functions	23
1.3.4	Governing Equations and Boundary Conditions	26
1.4	Lamé Parameters for Plates and Shells.	29
<b>2</b>	<b>Modified Fourier Series and Rayleigh-Ritz Method</b>	<b>37</b>
2.1	Modified Fourier Series	38
2.1.1	Traditional Fourier Series Solutions.	39
2.1.2	One-Dimensional Modified Fourier Series Solutions	43
2.1.3	Two-Dimensional Modified Fourier Series Solutions	48
2.2	Strong Form Solution Procedure	53
2.3	Rayleigh-Ritz Method (Weak Form Solution Procedure)	58
<b>3</b>	<b>Straight and Curved Beams</b>	<b>63</b>
3.1	Fundamental Equations of Thin Laminated Beams	64
3.1.1	Kinematic Relations	64
3.1.2	Stress-Strain Relations and Stress Resultants	65
3.1.3	Energy Functions	66

3.1.4	Governing Equations and Boundary Conditions . . . . .	68
3.2	Fundamental Equations of Thick Laminated Beams. . . . .	71
3.2.1	Kinematic Relations . . . . .	71
3.2.2	Stress-Strain Relations and Stress Resultants . . . . .	72
3.2.3	Energy Functions . . . . .	73
3.2.4	Governing Equations and Boundary Conditions . . . . .	74
3.3	Solution Procedures . . . . .	76
3.3.1	Strong Form Solution Procedure. . . . .	77
3.3.2	Weak Form Solution Procedure (Rayleigh-Ritz Procedure) . . . . .	80
3.4	Laminated Beams with General Boundary Conditions . . . . .	83
3.4.1	Convergence Studies and Result Verification . . . . .	83
3.4.2	Effects of Shear Deformation and Rotary Inertia. . . . .	85
3.4.3	Effects of the Deepness Term $(1 + z/R)$ . . . . .	87
3.4.4	Isotropic and Laminated Beams with General Boundary Conditions. . . . .	90
4	<b>Plates</b> . . . . .	99
4.1	Fundamental Equations of Thin Laminated Rectangular Plates. . . . .	100
4.1.1	Kinematic Relations . . . . .	100
4.1.2	Stress-Strain Relations and Stress Resultants . . . . .	102
4.1.3	Energy Functions . . . . .	103
4.1.4	Governing Equations and Boundary Conditions . . . . .	105
4.2	Fundamental Equations of Thick Laminated Rectangular Plates. . . . .	107
4.2.1	Kinematic Relations . . . . .	108
4.2.2	Stress-Strain Relations and Stress Resultants . . . . .	108
4.2.3	Energy Functions . . . . .	109
4.2.4	Governing Equations and Boundary Conditions . . . . .	111
4.3	Vibration of Laminated Rectangular Plates. . . . .	113
4.3.1	Convergence Studies and Result Verification . . . . .	116
4.3.2	Laminated Rectangular Plates with Arbitrary Classical Boundary Conditions . . . . .	117
4.3.3	Laminated Rectangular Plates with Elastic Boundary Conditions. . . . .	122
4.3.4	Laminated Rectangular Plates with Internal Line Supports. . . . .	125
4.4	Fundamental Equations of Laminated Sectorial, Annular and Circular Plates . . . . .	131
4.4.1	Fundamental Equations of Thin Laminated Sectorial, Annular and Circular Plates . . . . .	134
4.4.2	Fundamental Equations of Thick Laminated Sectorial, Annular and Circular Plates . . . . .	137

4.5	Vibration of Laminated Sectorial, Annular and Circular Plates . . . . .	139
4.5.1	Vibration of Laminated Annular and Circular Plates . . . . .	139
4.5.2	Vibration of Laminated Sectorial Plates . . . . .	144
5	<b>Cylindrical Shells</b> . . . . .	153
5.1	Fundamental Equations of Thin Laminated Cylindrical Shells . . . . .	156
5.1.1	Kinematic Relations . . . . .	156
5.1.2	Stress-Strain Relations and Stress Resultants . . . . .	157
5.1.3	Energy Functions . . . . .	158
5.1.4	Governing Equations and Boundary Conditions . . . . .	159
5.2	Fundamental Equations of Thick Laminated Cylindrical Shells . . . . .	162
5.2.1	Kinematic Relations . . . . .	162
5.2.2	Stress-Strain Relations and Stress Resultants . . . . .	163
5.2.3	Energy Functions . . . . .	165
5.2.4	Governing Equations and Boundary Conditions . . . . .	167
5.3	Vibration of Laminated Closed Cylindrical Shells . . . . .	169
5.3.1	Convergence Studies and Result Verification . . . . .	172
5.3.2	Effects of Shear Deformation and Rotary Inertia . . . . .	175
5.3.3	Laminated Closed Cylindrical Shells with General End Conditions . . . . .	177
5.3.4	Laminated Closed Cylindrical Shells with Intermediate Ring Supports . . . . .	184
5.4	Vibration of Laminated Open Cylindrical Shells . . . . .	188
5.4.1	Convergence Studies and Result Verification . . . . .	192
5.4.2	Laminated Open Cylindrical Shells with General End Conditions . . . . .	193
6	<b>Conical Shells</b> . . . . .	199
6.1	Fundamental Equations of Thin Laminated Conical Shells . . . . .	200
6.1.1	Kinematic Relations . . . . .	201
6.1.2	Stress-Strain Relations and Stress Resultants . . . . .	201
6.1.3	Energy Functions . . . . .	202
6.1.4	Governing Equations and Boundary Conditions . . . . .	202
6.2	Fundamental Equations of Thick Laminated Conical Shells . . . . .	207
6.2.1	Kinematic Relations . . . . .	208
6.2.2	Stress-Strain Relations and Stress Resultants . . . . .	209
6.2.3	Energy Functions . . . . .	210
6.2.4	Governing Equations and Boundary Conditions . . . . .	211
6.3	Vibration of Laminated Closed Conical Shells . . . . .	215
6.3.1	Convergence Studies and Result Verification . . . . .	217

6.3.2	Laminated Closed Conical Shells with General Boundary Conditions. . . . .	219
6.4	Vibration of Laminated Open Conical Shells . . . . .	225
6.4.1	Convergence Studies and Result Verification . . . . .	227
6.4.2	Laminated Open Conical Shells with General Boundary Conditions. . . . .	228
<b>7</b>	<b>Spherical Shells . . . . .</b>	<b>235</b>
7.1	Fundamental Equations of Thin Laminated Spherical Shells . . .	237
7.1.1	Kinematic Relations . . . . .	237
7.1.2	Stress-Strain Relations and Stress Resultants . . . . .	238
7.1.3	Energy Functions . . . . .	238
7.1.4	Governing Equations and Boundary Conditions . . . . .	239
7.2	Fundamental Equations of Thick Laminated Spherical Shells. . .	241
7.2.1	Kinematic Relations . . . . .	241
7.2.2	Stress-Strain Relations and Stress Resultants . . . . .	242
7.2.3	Energy Functions . . . . .	243
7.2.4	Governing Equations and Boundary Conditions . . . . .	245
7.3	Vibration of Laminated Closed Spherical Shells . . . . .	250
7.3.1	Convergence Studies and Result Verification . . . . .	251
7.3.2	Closed Laminated Spherical Shells with General Boundary Conditions. . . . .	252
7.4	Vibration of Laminated Open Spherical Shells . . . . .	257
7.4.1	Convergence Studies and Result Verification . . . . .	260
7.4.2	Laminated Open Spherical Shells with General Boundary Conditions. . . . .	263
<b>8</b>	<b>Shallow Shells. . . . .</b>	<b>271</b>
8.1	Fundamental Equations of Thin Laminated Shallow Shells. . . .	273
8.1.1	Kinematic Relations . . . . .	274
8.1.2	Stress-Strain Relations and Stress Resultants . . . . .	275
8.1.3	Energy Functions . . . . .	275
8.1.4	Governing Equations and Boundary Conditions . . . . .	276
8.2	Fundamental Equations of Thick Laminated Shallow Shells . . .	279
8.2.1	Kinematic Relations . . . . .	279
8.2.2	Stress-Strain Relations and Stress Resultants . . . . .	280
8.2.3	Energy Functions . . . . .	280
8.2.4	Governing Equations and Boundary Conditions . . . . .	281
8.3	Vibration of Laminated Shallow Shells . . . . .	284
8.3.1	Convergence Studies and Result Verification . . . . .	286
8.3.2	Laminated Shallow Shells with General Boundary Conditions. . . . .	288
	<b>References and Further Reading . . . . .</b>	<b>305</b>