

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	The nonstationary ideal on $\omega_1$ . . . . .	2
1.2	The partial order $\mathbb{P}_{\max}$ . . . . .	6
1.3	$\mathbb{P}_{\max}$ variations . . . . .	10
1.4	Extensions of inner models beyond $L(\mathbb{R})$ . . . . .	13
1.5	Concluding remarks – the view from Berlin in 1999 . . . . .	15
1.6	The view from Heidelberg in 2010 . . . . .	18
<b>2</b>	<b>Preliminaries</b>	21
2.1	Weakly homogeneous trees and scales . . . . .	21
2.2	Generic absoluteness . . . . .	31
2.3	The stationary tower . . . . .	34
2.4	Forcing Axioms . . . . .	36
2.5	Reflection Principles . . . . .	41
2.6	Generic ideals . . . . .	43
<b>3</b>	<b>The nonstationary ideal</b>	51
3.1	The nonstationary ideal and $\delta_2^1$ . . . . .	51
3.2	The nonstationary ideal and CH . . . . .	108
<b>4</b>	<b>The <math>\mathbb{P}_{\max}</math>-extension</b>	116
4.1	Iterable structures . . . . .	116
4.2	The partial order $\mathbb{P}_{\max}$ . . . . .	136
<b>5</b>	<b>Applications</b>	184
5.1	The sentence $\phi_{AC}$ . . . . .	184
5.2	Martin’s Maximum and $\phi_{AC}$ . . . . .	187
5.3	The sentence $\psi_{AC}$ . . . . .	192
5.4	The stationary tower and $\mathbb{P}_{\max}$ . . . . .	199
5.5	$\mathbb{P}_{\max}^*$ . . . . .	221
5.6	$\mathbb{P}_{\max}^0$ . . . . .	232
5.7	The Axiom $(*)$ . . . . .	238
5.8	Homogeneity properties of $\mathcal{P}(\omega_1)/\mathcal{I}_{NS}$ . . . . .	274
<b>6</b>	<b><math>\mathbb{P}_{\max}</math> variations</b>	287
6.1	${}^2\mathbb{P}_{\max}$ . . . . .	288
6.2	Variations for obtaining $\omega_1$ -dense ideals . . . . .	306
6.2.1	$\mathbb{Q}_{\max}$ . . . . .	306
6.2.2	$\mathbb{Q}_{\max}^*$ . . . . .	334

6.2.3	${}^2\mathbb{Q}_{\max}$ . . . . .	370
6.2.4	Weak Kurepa trees and $\mathbb{Q}_{\max}$ . . . . .	377
6.2.5	$\text{KT}\mathbb{Q}_{\max}$ . . . . .	383
6.2.6	Null sets and the nonstationary ideal . . . . .	403
6.3	Nonregular ultrafilters on $\omega_1$ . . . . .	421
<b>7</b>	<b>Conditional variations</b> . . . . .	<b>426</b>
7.1	Suslin trees . . . . .	426
7.2	The Borel Conjecture . . . . .	441
<b>8</b>	<b>principles for <math>\omega_1</math></b> . . . . .	<b>493</b>
8.1	Condensation Principles . . . . .	496
8.2	$\mathbb{P}_{\max}^{\clubsuit\text{NS}}$ . . . . .	501
8.3	The principles, $\clubsuit_{\text{NS}}^+$ and $\clubsuit_{\text{NS}}^{++}$ . . . . .	577
<b>9</b>	<b>Extensions of <math>L(\Gamma, \mathbb{R})</math></b> . . . . .	<b>609</b>
9.1	$\text{AD}^+$ . . . . .	610
9.2	The $\mathbb{P}_{\max}$ -extension of $L(\Gamma, \mathbb{R})$ . . . . .	617
9.2.1	The basic analysis . . . . .	618
9.2.2	Martin's Maximum $^{++}(c)$ . . . . .	622
9.3	The $\mathbb{Q}_{\max}$ -extension of $L(\Gamma, \mathbb{R})$ . . . . .	633
9.4	Chang's Conjecture . . . . .	637
9.5	Weak and Strong Reflection Principles . . . . .	651
9.6	Strong Chang's Conjecture . . . . .	667
9.7	Ideals on $\omega_2$ . . . . .	683
<b>10</b>	<b>Further results</b> . . . . .	<b>694</b>
10.1	Forcing notions and large cardinals . . . . .	694
10.2	Coding into $L(\mathcal{P}(\omega_1))$ . . . . .	701
10.2.1	Coding by sets, $\tilde{S}$ . . . . .	703
10.2.2	$\mathbb{Q}_{\max}^{(X)}$ . . . . .	708
10.2.3	$\mathbb{P}_{\max}^{(\emptyset)}$ . . . . .	739
10.2.4	$\mathbb{P}_{\max}^{(\emptyset, B)}$ . . . . .	768
10.3	Bounded forms of Martin's Maximum . . . . .	784
10.4	$\Omega$ -logic . . . . .	807
10.5	$\Omega$ -logic and the Continuum Hypothesis . . . . .	813
10.6	The Axiom $(*)^+$ . . . . .	827
10.7	The Effective Singular Cardinals Hypothesis . . . . .	835
<b>11</b>	<b>Questions</b> . . . . .	<b>840</b>
	<b>Bibliography</b> . . . . .	<b>845</b>
	<b>Index</b> . . . . .	<b>849</b>