

Contents

Part I Surveys

1	Copula Theory: An Introduction	3
	Fabrizio Durante and Carlo Sempi	
1.1	Historical Introduction	3
1.1.1	Outline	6
1.2	Preliminaries on Random Variables and Distribution Functions ...	6
1.3	Copulas: Definitions and Basic Properties	9
1.4	Sklar's Theorem	12
1.5	Copulas and Random Vectors	14
1.6	Families of Copulas	15
1.6.1	Elliptical Copulas	16
1.6.2	Archimedean Copulas	17
1.6.3	EFGM Copulas	19
1.7	Constructions of Copulas	20
1.7.1	Copulas with Given Lower Dimensional Marginals	20
1.7.2	Copula-to-Copula Transformations	21
1.7.3	Geometric Constructions of Copulas	22
1.8	Copula Theory: What's the Future?	23
	References	24
2	Dynamic Modeling of Dependence in Finance via Copulae Between Stochastic Processes	33
	Tomasz R. Bielecki, Jacek Jakubowski and Mariusz Niewęłowski	
2.1	Introduction	33
2.2	Lévy Copulae	35
2.3	Semimartingale Copulae	39
2.3.1	Copulae for Special Semimartingales	39
2.3.2	Consistent Semimartingale Copulae	48
2.4	Markov Copulae	54
2.4.1	Consistent Markov Processes	55

2.4.2	Markov Copulae: Generator Approach	57
2.4.3	Markov Copulae: Symbolic Approach	63
2.5	Applications in Finance	69
2.5.1	Pricing Rating-Triggered Step-Up Bonds via Simulation	70
2.5.2	Model Calibration and Pricing	72
	References	75
3	Copula Estimation	77
	Barbara Choroś, Rustam Ibragimov and Elena Permiakova	
3.1	Introduction	77
3.2	Copula Estimation: Random Samples with Dependent Marginals	78
3.2.1	Parametric Models: Maximum Likelihood Methods and Inference from Likelihoods for Margins	78
3.2.2	Semiparametric Estimation	80
3.2.3	Nonparametric Inference and Empirical Copula Processes	81
3.3	Copula-Based Time Series and Their Estimation	82
3.3.1	Copula-Based Characterizations for (Higher-Order) Markov Processes	82
3.3.2	Parametric and Semiparametric Copula Estimation Methods for Markov Processes	83
3.3.3	Nonparametric Copula Inference for Time Series	84
3.3.4	Dependence Properties of Copula-Based Time Series	85
3.4	Further Copula Inference Methods	86
3.5	Empirical Applications	87
	References	89
4	Pair-Copula Constructions of Multivariate Copulas	93
	Claudia Czado	
4.1	Introduction	93
4.2	Pair Copula Constructions of D-Vine, Canonical and Regular Vine Distributions	94
4.2.1	Pair-Copula Constructions of D-Vine and Canonical Vine Distributions	94
4.2.2	Regular Vines Distributions and Copulas	96
4.3	Estimation Methods for Regular Vine Copulas	100
4.4	Model Selection Among Vine Specifications	103
4.5	Applications of Vine Distributions	105
4.6	Summary and Open Problems	106
	References	107
5	Risk Aggregation	111
	Paul Embrechts and Giovanni Puccetti	
5.1	Motivations and Preliminaries	112
5.1.1	The Mathematical Framework	112
5.2	Bounds for Functions of Risks: The Coupling-Dual Approach	113
5.2.1	Application 1: Bounding Value-at-Risk	115

5.2.2	Application 2: Supermodular Functions	119
5.3	The Calculation of the Distribution of the Sum of Risks	120
5.3.1	Open Problems	124
	References	125
6	Extreme-Value Copulas	127
	Gordon Gudendorf and Johan Segers	
6.1	Introduction	127
6.2	Foundations	128
6.3	Parametric Models	131
6.3.1	Logistic Model or Gumbel–Hougaard Copula	132
6.3.2	Negative Logistic Model or Galambos Copula	132
6.3.3	Hüsler–Reiss Model	133
6.3.4	The t-EV Copula	134
6.4	Dependence Coefficients	134
6.5	Estimation	136
6.5.1	Parametric Estimation	137
6.5.2	Nonparametric Estimation	138
6.6	Further Reading	140
	References	141
7	Construction and Sampling of Nested Archimedean Copulas	147
	Marius Hofert	
7.1	Introduction	147
7.2	Nested Archimedean Copulas	149
7.3	A Sufficient Nesting Condition	151
7.4	Construction of Nested Archimedean Copulas	153
7.5	Sampling Nested Archimedean Copulas	155
7.6	Conclusion	159
	References	159
8	Tail Behaviour of Copulas	161
	Piotr Jaworski	
8.1	Introduction	161
8.2	Tail Expansions of Copulas	163
8.2.1	Characterization and Properties of Leading Parts	167
8.2.2	Relatively Invariant Measures on $[0, \infty)^n$	168
8.3	Examples of Tail Expansions	169
8.3.1	Homogeneous Copulas	169
8.3.2	Diagonal Copulas	169
8.3.3	Absolutely Continuous Copulas	171
8.3.4	Archimedean Copulas	172
8.3.5	Multivariate Extreme Value Copulas	177
8.4	Applications	178
8.4.1	Tail Conditional Copulas	178
8.4.2	Extreme Value Copulas of a Given Copula	180

8.4.3	Regularly Varying Random Vectors with a Given Copula	181
8.4.4	Value at Risk	182
	References	185
9	Copulae in Reliability Theory (Order Statistics, Coherent Systems)	187
	Tomasz Rychlik	
9.1	Coherent Systems	187
9.2	Signatures	189
9.2.1	Components with i.i.d. Lifetimes	189
9.2.2	Mixed Systems	190
9.2.3	Components with Exchangeable Lifetimes	192
9.3	Bounds for Exchangeable Lifetime Components	194
9.3.1	Distribution Bounds	194
9.3.2	Expectation Bounds	196
9.4	Characterizations of k -Out-of- n System Lifetime Distributions	198
9.4.1	General Copula Joint Distribution	199
9.4.2	Absolute Continuous Copula Joint Distribution	200
9.4.3	Variance Bounds	203
9.5	Final Remarks	205
	References	206
10	Copula-Based Measures of Multivariate Association	209
	Friedrich Schmid, Rafael Schmidt, Thomas Blumentritt, Sandra Gaißer and Martin Ruppert	
10.1	Introduction and Definitions	209
10.2	Aspects of Multivariate Association	212
10.3	Multivariate Generalizations of Spearman's Rho, Kendall's Tau, Blomqvist's Beta, and Gini's Gamma	215
10.3.1	Spearman's Rho	215
10.3.2	Kendall's Tau	217
10.3.3	Blomqvist's Beta	219
10.3.4	Gini's Gamma	220
10.4	Information-Based Measures of Multivariate Association	221
10.5	Measures of Multivariate Association Based on L_p -Distances	224
10.5.1	Φ^2 as a L_2 -Distance-Based Measure	225
10.5.2	σ as a L_1 -Distance-Based Measure	227
10.5.3	κ as a L_∞ -Distance-Based Measure	227
10.6	Multivariate Tail Dependence	228
	References	232
11	Semi-Copulas and Interpretations of Coincidences Between Stochastic Dependence and Ageing	237
	Fabio Spizzichino	
11.1	Introduction	237
11.2	Univariate Ageing and Dependence Properties of Archimedean Semi-Copulas	240

11.3 Dependence and Univariate Ageing in Schur-Constant Models ... 243

11.4 Level Curves, B functions, Duality, and Interpretation of
Coincidence Between Ageing and Dependence 247

11.5 Summary and Concluding Remarks 251

References 252

Part II Contributed Papers

**12 A Copula-Based Model for Spatial and Temporal Dependence of
Equity Markets 257**
Umberto Cherubini, Fabio Gobbi, Sabrina Mulinacci and Silvia
Romagnoli

12.1 Introduction 257

12.2 A market Model in Discrete Time 258

12.3 The Martingale Property 259

12.4 Applications 261

12.4.1 Multivariate Digital Options 261

12.4.2 Basket and Spread Options 263

References 264

**13 Nonparametric and Semiparametric Bivariate Modeling of
Petrophysical Porosity-Permeability Dependence from Well Log
Data 267**
Arturo Erdely and Martin Diaz-Viera

13.1 Introduction 267

13.2 Methodology 268

13.3 Data Analysis 271

13.4 Final Remarks 276

References 278

14 Testing Under the Extended Koziol-Green Model 279
Auguste Gaddah and Roel Braekers

14.1 Introduction 279

14.2 Asymptotic Results 282

14.3 Test Statistics 285

14.4 Data Example: Survival with Malignant Melanoma 286

References 288

**15 Parameter Estimation and Application of the Multivariate Skew
 t -Copula 289**
Tõnu Kollo and Gaida Pettere

15.1 Introduction 289

15.2 Preliminary Notions and Notation 290

15.3 Construction of a Skew t -Copula 292

15.4 Parameter Estimation 293

15.5 Simulation 295

15.6	Application	296
	References	298
16	On Analytical Similarities of Archimedean and Exchangeable Marshall-Olkin Copulas	299
	Jan-Frederik Mai and Matthias Scherer	
16.1	Introduction	299
16.2	Complete Monotonicity and d -Monotonicity	301
16.2.1	Definitions and Examples	301
16.2.2	Probabilistic Interpretations	302
16.2.3	d -Monotonicity	303
16.3	Probabilistic Models and Sampling	305
16.3.1	The Completely Monotone Case	305
16.3.2	The Proper d -Monotone Case	306
	References	308
17	Relationships Between Archimedean Copulas and Morgenstern Utility Functions	311
	Jaap Spreeuw	
17.1	Introduction	311
17.2	Archimedean Copulas	312
17.3	Utility Functions	312
17.4	Relationships Between Properties of Utility Functions and Properties of Generators	315
17.5	Examples	318
17.5.1	Classical Cases	318
17.5.2	The HARA Family	319
17.5.3	The Expo Power Utility	319
17.5.4	Other Examples of Decreasing Absolute Risk Aversion (DARA) as in Pratt [9]	319
17.6	Conclusion	321
	References	321
Index		323