

# Contents

|                                   |                                                      |     |
|-----------------------------------|------------------------------------------------------|-----|
| <i>Introduction</i>               |                                                      | 1   |
| In.1                              | A Perspective on Hilbert's Programs                  | 3   |
| In.2                              | Milestones                                           | 29  |
| <i>I Mathematical roots</i>       |                                                      | 33  |
| I.1                               | Dedekind's analysis of number: systems and axioms    | 35  |
| I.2                               | Methods for real arithmetic                          | 73  |
| I.3                               | Hilbert's programs: 1917–1922                        | 91  |
| <i>II Analyses</i>                |                                                      |     |
|                                   | <i>Historical</i>                                    | 129 |
| II.1                              | Finitist proof theory: 1922–1934                     | 131 |
| II.2                              | After Königsberg                                     | 145 |
| II.3                              | In the shadow of incompleteness: Hilbert and Gentzen | 155 |
| II.4                              | Gödel at Zilsel's                                    | 193 |
| II.5                              | Hilbert and Bernays: 1939                            | 215 |
|                                   | <i>Systematical</i>                                  | 227 |
| II.6                              | Foundations for analysis and proof theory            | 229 |
| II.7                              | Reductions of theories for analysis                  | 263 |
| II.8                              | Hilbert's program sixty years later                  | 281 |
| II.9                              | On reverse mathematics                               | 291 |
| II.10                             | Relative consistency and accessible domains          | 299 |
| <i>III Philosophical horizons</i> |                                                      | 327 |
| III.1                             | Aspects of mathematical experience                   | 329 |

|       |                                                                              |     |
|-------|------------------------------------------------------------------------------|-----|
| III.2 | Beyond Hilbert's reach?                                                      | 345 |
| III.3 | Searching for proofs (and uncovering capacities<br>of the mathematical mind) | 377 |
|       | Bibliography                                                                 | 403 |
|       | Index                                                                        | 435 |