

Contents

1	Introduction	1
1.1	High-Order Numerical Methods on Unstructured Grids	3
1.2	Applications and Challenges for Unstructured High-Order Methods	5
1.2.1	Computational Cost and Storage	6
1.2.2	Boundary Treatment	6
1.2.3	Shock Capturing Technique	7
1.2.4	Efficient Time Integration Strategy	9
1.3	The Target of This Thesis	9
	References	11
2	High-Order Finite Volume Method for the Compressible Flows	15
2.1	The Framework of the Finite Volume Method	15
2.2	The Time Integration Schemes	17
2.3	High-Order <i>K</i> -Exact Reconstruction	21
2.4	Evaluation of Viscous Flux in High-Order Accuracy	23
2.4.1	The Viscous Flux Computation	23
2.4.2	On the Implementation of Viscous Boundary Conditions	24
2.5	Parallelization for Large-Scale Computation	25
2.6	Numerical Tests	26
2.6.1	Accuracy Validation	27
2.6.2	The Flat Plate Boundary Layer	30
2.6.3	The Subsonic Flow Around NACA0012 Airfoil	30
2.6.4	Sound Field Generated by the Flow Around a Circular Cylinder	32
2.6.5	The Steady Viscous Flow Around the Sphere	34
2.7	Conclusions	35
	References	35

3 Accuracy Preserving Limiters for High-Order Finite Volume Methods	37
3.1 The Limiters on Unstructured Grids and the Methodology of Constructing Limiters in This Chapter	39
3.2 The Secondary Reconstruction	40
3.3 The Quadrature-Free K-Exact WENO Limiters	43
3.3.1 The Stencils for the Secondary Reconstruction	44
3.3.2 Quadrature-Free Nonoscillation WENO Reconstruction in Characteristic Space	45
3.4 The WBAP Limiters	48
3.4.1 The Introduction of WBAP Functions	49
3.4.2 The Mechanism for Controlling the Numerical Oscillations	53
3.4.3 The Successive Limiting Procedure	56
3.5 The Group Weighted Limiters	62
3.6 Summary and Comparisons on These Three Limiters	65
3.7 The Problem-Independent Shock Detector	66
3.8 Numerical Accuracy Test	67
3.8.1 Isentropic Vortex Problem	68
3.8.2 The Constructed Exact Solution of Three Dimensional Euler Equations	68
3.9 Numerical Tests	69
3.9.1 Rotation of Slotted Cylinder	70
3.9.2 Shock Tube Problems	72
3.9.3 Shu-Osher Problem	75
3.9.4 Double Mach Reflection Problem	77
3.9.5 A Mach 3 Wind Tunnel with a Step	78
3.9.6 Transonic Flow Around NACA0012 Airfoil	79
3.9.7 Shock Wave Impingement on a Spatially Evolving Mixing Layer	84
3.9.8 Viscous Shock Tube Problem	86
3.9.9 Explosion Problems on Two and Three Dimensions	87
3.10 Conclusions	89
References	89
4 Mixed Element and Curved Boundary Treatment	93
4.1 The Applications of Isoparametric Transformation on Mixed Elements and Curved Boundaries	95
4.1.1 The Isoparametric Transformation on Mixed Elements	95
4.1.2 The Isoparametric Transformation on Curved Elements	98

4.2	The Nonoscillatory High-Order FVM in Mixed Grids	101
4.2.1	The k-Exact Reconstruction and Secondary Reconstruction.	102
4.2.2	The k-Exact WENO Limiter	103
4.2.3	The WBAP Limiter Based on the Successive Limiting Procedure	105
4.3	The Curved Boundary Treatment	105
4.4	Numerical Tests.	107
4.4.1	Supersonic Vortex	107
4.4.2	The Supersonic Viscous Flow Around the NACA0012 Airfoil	107
4.4.3	The Three-Dimensional Constructed Solutions	109
4.4.4	Lax Shock Tube Problem	110
4.4.5	Transonic Flow Around ONERA M6 Wing	112
4.4.6	The Unsteady Flow Around the Sphere	114
4.4.7	The Supersonic Flow Around the Sphere	116
4.5	Conclusions.	118
	References	120
5	The Discontinuous Galerkin Method.	121
5.1	Framework of the Discontinuous Galerkin Methods	121
5.2	The Calculation of Viscous Terms	123
5.3	The Construction of the Multidimensional Limiters	125
5.3.1	The Secondary Reconstruction	125
5.3.2	The WENO Limiting Procedure in Characteristic Space	127
5.4	Numerical Tests.	129
5.4.1	Accuracy Tests	130
5.4.2	Shock Capturing Capability	133
5.4.3	Double Mach Reflection.	134
5.4.4	Mach 3 Wind Tunnel with a Step	135
5.4.5	Viscous Shock Tube Problem	137
5.4.6	The Unsteady Flow Around the Sphere	137
5.5	Conclusions.	139
	References	139
6	Conclusions and Prospects	141
	Appendix A: The Derivation of the Two Limits of WBAP Limiters. . .	143
	Appendix B: The Proof of WBAP Lemma 1.	147