

# Contents

|                                                                     |             |
|---------------------------------------------------------------------|-------------|
| <b>Preface</b>                                                      | <b>xi</b>   |
| <b>Introduction</b>                                                 | <b>xiii</b> |
| <b>1 Arguing about Classical Logic</b>                              | <b>1</b>    |
| 1.1 Logical Theories . . . . .                                      | 1           |
| 1.1.1 The Bearers of Logical Properties . . . . .                   | 2           |
| 1.1.2 Total and Partial Logical Theories . . . . .                  | 3           |
| 1.1.3 The Shape of a Logical Theory . . . . .                       | 4           |
| 1.2 Classical and Intuitionistic Logic . . . . .                    | 6           |
| 1.2.1 The Formal Syntactic Part . . . . .                           | 6           |
| 1.2.2 A Model-Theoretic Logical Theory . . . . .                    | 8           |
| 1.2.3 Two Derivational Logical Theories . . . . .                   | 10          |
| 1.2.4 The Transformational Part . . . . .                           | 13          |
| 1.2.5 Logically Valid Rules . . . . .                               | 16          |
| 1.3 Arguing Against a Logical Theory . . . . .                      | 19          |
| 1.3.1 Types of Sentences of the Language of $\mathcal{T}$ . . . . . | 19          |
| 1.3.2 Four Ways of Arguing Against a Logical Theory . . . . .       | 21          |
| 1.4 Relevance . . . . .                                             | 24          |
| 1.4.1 Conditionality, Implication, and Validity . . . . .           | 25          |
| 1.4.2 Relevantism and Classical Mathematics . . . . .               | 27          |
| 1.5 Empty Singular Terms . . . . .                                  | 29          |
| 1.5.1 Theories of Free Logic . . . . .                              | 30          |
| 1.5.2 Free Logic and Non-Standard Classical Logic . . . . .         | 31          |
| <b>2 A Purely Verbal Dispute?</b>                                   | <b>35</b>   |
| 2.1 Mathematical Languages . . . . .                                | 36          |
| 2.1.1 Natural Mathematical Languages . . . . .                      | 36          |
| 2.1.2 Formal Mathematical Languages . . . . .                       | 38          |
| 2.2 Logical Vocabulary . . . . .                                    | 39          |
| 2.2.1 Inclusive Monism and the Burden of Proof . . . . .            | 41          |
| 2.2.2 Meta-Linguistic Uniqueness Results . . . . .                  | 42          |
| 2.2.3 Rule-Based Uniqueness Results . . . . .                       | 46          |
| 2.2.4 Philosophical Consequences of Uniqueness . . . . .            | 51          |
| 2.3 Mathematical Vocabulary . . . . .                               | 57          |
| 2.3.1 Theories About Numbers . . . . .                              | 58          |
| 2.3.2 Theories About Sets of Numbers . . . . .                      | 63          |

|       |                                                            |     |
|-------|------------------------------------------------------------|-----|
| 2.3.3 | Theories About Sequences of Numbers . . . . .              | 66  |
| 2.3.4 | Second-Order Theories . . . . .                            | 68  |
| 2.4   | The Simplicity of Understanding . . . . .                  | 72  |
| 2.4.1 | An Argument Against Shared Understanding . . . . .         | 73  |
| 2.4.2 | A Causal Theory of Understanding . . . . .                 | 75  |
| 2.4.3 | Low Requirements for Understanding . . . . .               | 77  |
| 3     | Counterexamples to Classical Logic . . . . .               | 81  |
| 3.1   | A Mathematical Conflict . . . . .                          | 82  |
| 3.1.1 | The Uniformity Principle (UP) . . . . .                    | 82  |
| 3.1.2 | The Continuity Principle (CP) . . . . .                    | 84  |
| 3.2   | Justifications of UP and CP . . . . .                      | 86  |
| 3.2.1 | A Justification of UP . . . . .                            | 87  |
| 3.2.2 | A Justification of CP . . . . .                            | 96  |
| 3.3   | Weak Counterexamples . . . . .                             | 100 |
| 3.3.1 | What are Weak Counterexamples? . . . . .                   | 101 |
| 3.3.2 | Establishing Weak Counterexamples . . . . .                | 103 |
| 3.4   | A Logical Basis of Mathematics? . . . . .                  | 106 |
| 3.4.1 | Brouwerian Mathematics . . . . .                           | 107 |
| 3.4.2 | The Normativity of Logic and Mathematics . . . . .         | 109 |
| 3.4.3 | Prawitz' Account of Correct Inferences . . . . .           | 114 |
| 4     | The Manifestation Argument . . . . .                       | 121 |
| 4.1   | A Sketch of the Manifestation Argument . . . . .           | 122 |
| 4.1.1 | A Basic Version of the Argument . . . . .                  | 123 |
| 4.1.2 | Three Variants of the Argument . . . . .                   | 126 |
| 4.2   | Truth Conditional Semantics . . . . .                      | 131 |
| 4.2.1 | Preliminaries . . . . .                                    | 132 |
| 4.2.2 | A Problem for Truth Conditional Semantics . . . . .        | 133 |
| 4.2.3 | Truth-Theory . . . . .                                     | 134 |
| 4.2.4 | The Semantic Significance of Theories of Truth . . . . .   | 138 |
| 4.2.5 | Refined Truth-Theory . . . . .                             | 139 |
| 4.3   | Understanding and Knowledge . . . . .                      | 142 |
| 4.3.1 | Ideal Understanding . . . . .                              | 142 |
| 4.3.2 | Understanding and Knowledge of Truth Conditions . . . . .  | 146 |
| 4.4   | Understanding and Use . . . . .                            | 149 |
| 4.4.1 | Dummettian Accounts of Understanding . . . . .             | 150 |
| 4.4.2 | Kinds of Manifesting Behaviour . . . . .                   | 154 |
| 4.4.3 | Descriptions of Manifesting Behaviour . . . . .            | 155 |
| 4.4.4 | Manifestability of Knowledge of Truth Conditions . . . . . | 156 |
| 4.5   | The Simplicity of Manifestability . . . . .                | 160 |
| 4.5.1 | Compositional Manifestation . . . . .                      | 161 |
| 4.5.2 | Manifestation and Epistemic Constraints . . . . .          | 167 |
| 5     | Proof-Theoretic Arguments . . . . .                        | 174 |
| 5.1   | Proof-Theoretic Preliminaries . . . . .                    | 175 |
| 5.1.1 | General Proof-Theoretic Concepts . . . . .                 | 175 |

|       |                                                            |     |
|-------|------------------------------------------------------------|-----|
| 5.1.2 | Calculi of Natural Deduction . . . . .                     | 179 |
| 5.1.3 | Schemata for Rules and Standard Calculi . . . . .          | 183 |
| 5.2   | Accounts of Meaning and Logical Frameworks . . . . .       | 186 |
| 5.2.1 | The General Form of Proof-Theoretic Arguments . . . . .    | 186 |
| 5.2.2 | Permissible Logical Frameworks . . . . .                   | 190 |
| 5.3   | Tennant's Proof-Theoretic Argument . . . . .               | 195 |
| 5.3.1 | Separability, Purity, Simplicity, and Directness . . . . . | 196 |
| 5.3.2 | Failure and Improvement . . . . .                          | 202 |
| 5.4   | Harmony . . . . .                                          | 206 |
| 5.4.1 | The Idea of Dummett's Proof-Theoretic Argument . . . . .   | 206 |
| 5.4.2 | Introduction and Elimination Rules . . . . .               | 209 |
| 5.4.3 | The Relation of Harmony . . . . .                          | 213 |
| 5.5   | Dummett's Proof-Theoretic Argument . . . . .               | 218 |
| 5.5.1 | The Fundamental Idea of Dummett's Argument . . . . .       | 220 |
| 5.5.2 | The First Version of Dummett's Argument . . . . .          | 224 |
| 5.5.3 | The Second Version of Dummett's Argument . . . . .         | 226 |
| 5.6   | Against Dummett's Proof-Theoretic Argument . . . . .       | 229 |
| 5.6.1 | Reflection and Conjugation . . . . .                       | 229 |
| 5.6.2 | Classical Logic and Non-Circular Classical Rules . . . . . | 236 |
| 6     | Conclusion . . . . .                                       | 240 |
| A     | Appendix to Chapter 2 . . . . .                            | 244 |
| A.1   | Arithmetical Uniqueness . . . . .                          | 244 |
| A.2   | Set-Theoretic Uniqueness . . . . .                         | 253 |
| A.3   | Function-Theoretic Uniqueness . . . . .                    | 255 |
|       | Bibliography . . . . .                                     | 257 |
|       | Index . . . . .                                            | 268 |