

# Contents

## Part I Mathematics of Distances

<b>1</b>	<b>General Definitions</b>	<b>3</b>
1.1	Basic Definitions	3
1.2	Main Distance-Related Notions	12
1.3	Metric Numerical Invariants	22
1.4	Main Mappings of Metric Spaces	36
1.5	General Distances	46
<b>2</b>	<b>Topological Spaces</b>	<b>63</b>
<b>3</b>	<b>Generalizations of Metric Spaces</b>	<b>71</b>
3.1	<i>m</i> -Tuple Generalizations of Metrics	71
3.2	Indefinite Metrics	73
3.3	Topological Generalizations	74
3.4	Beyond Numbers	78
<b>4</b>	<b>Metric Transforms</b>	<b>85</b>
4.1	Metrics on the Same Set	85
4.2	Metrics on Set Extensions	89
4.3	Metrics on Other Sets	92
<b>5</b>	<b>Metrics on Normed Structures</b>	<b>95</b>

## Part II Geometry and Distances

<b>6</b>	<b>Distances in Geometry</b>	<b>109</b>
6.1	Geodesic Geometry	109
6.2	Projective Geometry	115
6.3	Affine Geometry	121
6.4	Non-Euclidean Geometry	123

<b>7</b>	<b>Riemannian and Hermitian Metrics</b> .....	133
7.1	Riemannian Metrics and Generalizations .....	134
7.2	Riemannian Metrics in Information Theory .....	150
7.3	Hermitian Metrics and Generalizations .....	154
<b>8</b>	<b>Distances on Surfaces and Knots</b> .....	167
8.1	General Surface Metrics .....	167
8.2	Intrinsic Metrics on Surfaces .....	173
8.3	Distances on Knots .....	178
<b>9</b>	<b>Distances on Convex Bodies, Cones, and Simplicial Complexes</b> .....	181
9.1	Distances on Convex Bodies .....	181
9.2	Distances on Cones .....	187
9.3	Distances on Simplicial Complexes .....	190

### **Part III Distances in Classical Mathematics**

<b>10</b>	<b>Distances in Algebra</b> .....	197
10.1	Group Metrics .....	197
10.2	Metrics on Binary Relations .....	206
10.3	Metrics on Semilattices .....	208
<b>11</b>	<b>Distances on Strings and Permutations</b> .....	213
11.1	Distances on General Strings .....	214
11.2	Distances on Permutations .....	222
<b>12</b>	<b>Distances on Numbers, Polynomials, and Matrices</b> .....	227
12.1	Metrics on Numbers .....	227
12.2	Metrics on Polynomials .....	233
12.3	Metrics on Matrices .....	235
<b>13</b>	<b>Distances in Functional Analysis</b> .....	245
13.1	Metrics on Function Spaces .....	245
13.2	Metrics on Linear Operators .....	252
<b>14</b>	<b>Distances in Probability Theory</b> .....	257
14.1	Distances on Random Variables .....	258
14.2	Distances on Distribution Laws .....	259

### **Part IV Distances in Applied Mathematics**

<b>15</b>	<b>Distances in Graph Theory</b> .....	275
15.1	Distances on the Vertices of a Graph .....	276
15.2	Distance-Defined Graphs .....	284
15.3	Distances on Graphs .....	294
15.4	Distances on Trees .....	302

<b>16 Distances in Coding Theory</b>	309
16.1 Minimum Distance and Relatives	311
16.2 Main Coding Distances	314
<b>17 Distances and Similarities in Data Analysis</b>	323
17.1 Similarities and Distances for Numerical Data	325
17.2 Relatives of Euclidean Distance	328
17.3 Similarities and Distances for Binary Data	331
17.4 Correlation Similarities and Distances	335
<b>18 Distances in Systems and Mathematical Engineering</b>	341
18.1 Distances in State Transition and Dynamical Systems	342
18.2 Distances in Control Theory	347
18.3 Motion Planning Distances	349
18.4 MOEA Distances	354

## Part V Computer-Related Distances

<b>19 Distances on Real and Digital Planes</b>	359
19.1 Metrics on Real Plane	359
19.2 Digital Metrics	369
<b>20 Voronoi Diagram Distances</b>	377
20.1 Classical Voronoi Generation Distances	378
20.2 Plane Voronoi Generation Distances	380
20.3 Other Voronoi Generation Distances	383
<b>21 Image and Audio Distances</b>	387
21.1 Image Distances	387
21.2 Audio Distances	401
<b>22 Distances in Networks</b>	413
22.1 Scale-Free Networks	413
22.2 Network-Based Semantic Distances	418
22.3 Distances in Internet and Web	422

## Part VI Distances in Natural Sciences

<b>23 Distances in Biology</b>	431
23.1 Genetic Distances	435
23.2 Distances for DNA/RNA and Protein Data	447
23.3 Distances in Ecology, Biogeography, Ethology	458
23.4 Other Biological Distances	472
<b>24 Distances in Physics and Chemistry</b>	487
24.1 Distances in Physics	487
24.2 Distances in Chemistry and Crystallography	511

<b>25</b>	<b>Distances in Earth Science and Astronomy</b> .....	521
25.1	Distances in Geography .....	521
25.2	Distances in Geophysics .....	533
25.3	Distances in Astronomy .....	544
<b>26</b>	<b>Distances in Cosmology and Theory of Relativity</b> .....	561
26.1	Distances in Cosmology .....	561
26.2	Distances in Theory of Relativity .....	570
 <b>Part VII Real-World Distances</b>		
<b>27</b>	<b>Length Measures and Scales</b> .....	595
27.1	Length Scales .....	595
27.2	Orders of Magnitude for Length .....	604
<b>28</b>	<b>Distances in Applied Social Sciences</b> .....	609
28.1	Distances in Perception and Psychology .....	609
28.2	Distances in Economics and Human Geography .....	619
28.3	Distances in Sociology and Language .....	632
28.4	Distances in Philosophy, Religion and Art .....	642
<b>29</b>	<b>Other Distances</b> .....	661
29.1	Distances in Medicine, Anthropometry and Sport .....	661
29.2	Equipment Distances .....	678
29.3	Miscellany .....	692
<b>References</b> .....		701
<b>Index</b> .....		709