

# Contents

<b>1</b>	<b>Modelling Biological Neurons in Terms of Electrical Circuits</b>	<b>1</b>
1.1	The Hodgkin–Huxley Equations	1
1.1.1	Nernst’s Equation	3
1.1.2	The Goldman–Hodgkin–Katz Equation	4
1.2	Equivalent Circuits of the Cell’s Membrane	5
1.2.1	The Electric Equivalent	5
1.2.2	Membrane’s Time Constant	8
1.3	Describing Membrane’s Voltage Dynamics with Cable’s Equation	9
1.4	The Hodgkin–Huxley Model	12
1.4.1	Derivation of the Hodgkin–Huxley Model	12
1.4.2	Outline of the Hodgkin–Huxley Equations	13
1.5	The FitzHugh–Nagumo Model of Neurons	14
1.6	The Morris–Lecar Model of Neurons	14
1.7	Modelling Dendrites in Terms of Electrical Circuits	16
1.7.1	Dendrites	16
1.7.2	Computing Cable’s Equation in Neurons	18
1.7.3	The Infinite Length Cable	19
1.8	Ion Channels and Their Characteristics	20
1.8.1	Types of Ion Channels	20
1.8.2	Sodium Channels $\text{Na}^+$	22
1.8.3	Calcium Channels $\text{Ca}^{2+}$	22
1.8.4	Voltage-Gated Potassium Channels $\text{K}^+$	23
1.8.5	Voltage Sags	24
1.8.6	Currents and Concentration of Ions	24
1.8.7	Calcium-Dependent Channels	25
1.9	Conclusions	26

<b>2</b>	<b>Systems Theory for the Analysis of Biological Neuron Dynamics.....</b>	<b>27</b>
2.1	Characteristics of the Dynamics of Nonlinear Systems .....	27
2.2	Computation of Isoclines .....	32
2.3	Systems Theory and Neurodynamics.....	34
2.3.1	The Phase Diagram .....	34
2.3.2	Stability Analysis of Nonlinear Systems .....	34
2.3.3	Stability Analysis of the Morris–Lecar Nonlinear Model.....	37
2.4	Phase Diagrams and Equilibria of Neuronal Models .....	38
2.4.1	Phase Diagrams for Linear Dynamical Systems .....	38
2.4.2	Multiple Equilibria for Nonlinear Dynamical Systems ...	40
2.4.3	Limit Cycles .....	44
2.5	Bifurcations in Neuronal Dynamics .....	46
2.5.1	Bifurcations of Fixed Points of Biological Neuron Models .....	46
2.5.2	Saddle-Node Bifurcations of Fixed Points in a One-Dimensional System .....	46
2.5.3	Pitchfork Bifurcation of Fixed Points .....	47
2.5.4	The Hopf Bifurcation.....	49
2.6	Chaos in Neurons.....	51
2.6.1	Chaotic Dynamics in Neurons .....	51
2.6.2	Chaotic Dynamics in Associative Memories .....	51
2.7	Conclusions .....	51
<b>3</b>	<b>Bifurcations and Limit Cycles in Models of Biological Systems.....</b>	<b>53</b>
3.1	Outline .....	53
3.2	Generalization of the Routh–Hurwitz Criterion with Kharitonov’s Theorem.....	55
3.2.1	Application of the Routh Criterion to Systems with Parametric Uncertainty .....	55
3.2.2	An Application Example .....	56
3.3	Stages in Bifurcations Analysis.....	57
3.4	Bifurcation Analysis of the FitzHugh–Nagumo Neuron .....	58
3.4.1	Equilibria and Stability of the FitzHugh–Nagumo Neuron .....	58
3.4.2	Condition for the Appearance of Limit Cycles.....	60
3.5	Bifurcation Analysis of Circadian Oscillators .....	61
3.5.1	Fixed Points Bifurcation Analysis Using Kharitonov’s Theory.....	61
3.5.2	Method to Detect Hopf Bifurcations in the Circadian Cells.....	64
3.6	Feedback Control of Bifurcations .....	66
3.7	Simulation Tests .....	71
3.8	Conclusions .....	73

<b>4</b>	<b>Oscillatory Dynamics in Biological Neurons</b>	<b>75</b>
4.1	Neural Oscillators	75
4.2	Synaptic Channels	77
4.3	Dynamics of the Synapses	78
4.4	Study of the Glutamate Neurotransmitter	80
4.5	Study of the GABA Neurotransmitter	81
4.6	Short- and Long-Term Plasticity	82
4.7	Synchronization of Coupled FitzHugh–Nagumo Neurons Using Differential Flatness Theory	83
4.7.1	The Problem of Synchronization of Coupled Neural Oscillators	83
4.7.2	Coupled Neural Oscillators as Coordinators of Motion	85
4.8	Differential Flatness Theory	86
4.8.1	Definition of Differentially Flat Systems	86
4.8.2	Conditions for Applying Differential Flatness Theory	87
4.8.3	Transformation of the Neurons' Model into the Linear Canonical Form	88
4.9	Linearization of the FitzHugh–Nagumo Neuron	89
4.9.1	Linearization of the FitzHugh–Nagumo Model Using a Differential Geometric Approach	89
4.9.2	Linearization of the FitzHugh–Nagumo Model Using Differential Flatness Theory	91
4.10	Linearization of Coupled FitzHugh–Nagumo Neurons Using Differential Geometry	92
4.11	Linearization of Coupled FitzHugh–Nagumo Neurons Using Differential Flatness Theory	96
4.11.1	Differential Flatness of the Model of the Coupled Neurons	96
4.11.2	Linearization of the Coupled Neurons Using Differential Flatness Theory	97
4.12	State and Disturbances Estimation with the Derivative-Free Nonlinear Kalman Filter	98
4.12.1	Kalman and Extended Kalman Filtering	98
4.12.2	Design of a Disturbance Observer for the Model of the Coupled Neurons	100
4.12.3	Disturbances Compensation for the Model of the Coupled Neurons	102
4.13	Simulation Tests	102
4.14	Conclusions	105

<b>5</b>	<b>Synchronization of Circadian Neurons and Protein Synthesis Control</b>	<b>107</b>
5.1	Overview	107
5.2	Modelling of Circadian Oscillators Dynamics	109
5.2.1	The Functioning of the Circadian Oscillators	109
5.2.2	Mathematical Model of the Circadian Oscillator	111
5.3	Protein Synthesis Control Using Differential Geometry Methods	112
5.4	Protein Synthesis Control Using Differential Flatness Theory	114
5.4.1	Differential Flatness of the Circadian Oscillator	114
5.4.2	Transformation into a Canonical Form	115
5.5	Robust Synchronization of Coupled Circadian Oscillators Using Differential Geometry Methods	118
5.6	Robust Synchronization of Coupled Circadian Oscillators Using Differential Flatness Theory	120
5.7	State Estimation and Disturbances Compensation with the Derivative-Free Nonlinear Kalman Filter	124
5.8	Simulation Tests	126
5.9	Conclusions	129
<b>6</b>	<b>Wave Dynamics in the Transmission of Neural Signals</b>	<b>131</b>
6.1	Outline	131
6.2	Propagating Action Potentials	132
6.3	Dynamics of the Solution of the Wave PDE	133
6.4	Myelinated Axons and Discrete Diffusion	133
6.4.1	Solitons in Neuron's Dynamical Model	135
6.4.2	Comparison Between the Hodgkin–Huxley and the Soliton Model	136
6.5	Estimation of Nonlinear Wave Dynamics	137
6.6	Simulation Tests	141
6.6.1	Evaluation Experiments	141
6.6.2	Assessment of the Filter's Performance	145
6.7	Conclusions	147
<b>7</b>	<b>Stochastic Models of Biological Neuron Dynamics</b>	<b>149</b>
7.1	Outline	149
7.2	Wiener Process and Its Equivalence to Diffusion	150
7.2.1	Neurons' Dynamics Under Noise	150
7.2.2	Wiener Walk and Wiener Process	151
7.2.3	Outline of Wiener Process Properties	152
7.2.4	The Wiener Process Corresponds to a Diffusion PDE	153
7.2.5	Stochastic Integrals	153
7.2.6	Ito's Stochastic Differential Equation	153

7.3	Fokker–Planck’s Partial Differential Equation .....	154
7.3.1	The Fokker–Planck Equation .....	154
7.3.2	First Passage Time .....	155
7.3.3	Meaning of the First Passage Time .....	155
7.4	Stochastic Modelling of Ion Channels .....	156
7.5	Fokker–Planck Equation and the Integrate-and-Fire Neuron Model .....	157
7.5.1	The Integrate-and-Fire Neuron Model .....	157
7.5.2	Stochastic Integrate-and-Fire Neuron Model and the Fokker–Planck Equation .....	160
7.5.3	Rate of Firing for Neural Models .....	161
7.6	Stochasticity in Neural Dynamics and Relation to Quantum Mechanics .....	161
7.6.1	Basics of Quantum Mechanics .....	161
7.6.2	Schrödinger’s Equation with Non-zero Potential and Its Equivalence to Diffusion with Drift .....	163
7.6.3	Study of the QHO Model Through the Ornstein–Uhlenbeck Diffusion .....	164
7.6.4	Particle’s Motion Is a Generalization of Gradient Algorithms .....	166
7.7	Conclusions .....	167
<b>8</b>	<b>Synchronization of Stochastic Neural Oscillators Using Lyapunov Methods .....</b>	<b>169</b>
8.1	Representation of the Neurons’ Dynamics as Brownian Motion .....	169
8.2	Interacting Diffusing Particles as a Model of Neural Networks ...	171
8.2.1	Weights’ Equivalence to Brownian Particles .....	171
8.2.2	Stability Analysis for a Neural Model with Brownian Weights .....	173
8.3	Convergence of the Stochastic Weights to an Equilibrium .....	178
8.4	Conclusions .....	180
<b>9</b>	<b>Synchronization of Chaotic and Stochastic Neurons Using Differential Flatness Theory .....</b>	<b>181</b>
9.1	Chaotic Neural Oscillators .....	181
9.1.1	Models of Chaotic Oscillators .....	181
9.1.2	Differential Flatness of Chaotic Oscillators .....	182
9.2	Stabilization of Interacting Particles Which Are Modelled as Coupled Stochastic Oscillators .....	184
9.3	Some Examples on Flatness-Based Control of Coupled Oscillators .....	185
9.4	Flatness-Based Control for the Multi-Particle System .....	187
9.5	Simulation Tests .....	189
9.6	Conclusions .....	190

<b>10</b>	<b>Attractors in Associative Memories with Stochastic Weights</b>	191
10.1	Weights Learning in Associative Memories Is a Wiener Process	191
10.1.1	The Weights of Associative Memories Are Equivalent to Brownian Particles	191
10.1.2	Mean Value and Variance of the Brownian Weights	193
10.1.3	Learning Through Unitary Quantum Mechanical Operators	195
10.2	Attractors in QHO-Based Associative Memories	196
10.2.1	Decomposition of the Weight Matrix into a Superposition of Matrices	196
10.2.2	Evolution Between the Eigenvector Spaces via Unitary Rotations	199
10.2.3	Applications of the Stochastic Associative Memory Model	200
10.3	Attractors in Associative Memories with Stochastic Weights	200
10.4	Conclusions	206
<b>11</b>	<b>Spectral Analysis of Neural Models with Stochastic Weights</b>	207
11.1	Overview	207
11.2	Wavelet Basis Functions	208
11.2.1	Wavelet Frames	208
11.2.2	Dyadic Grid Scaling and Orthonormal Wavelet Transforms	209
11.2.3	The Scaling Function and the Multi-resolution Representation	210
11.2.4	Examples of Orthonormal Wavelets	211
11.2.5	The Haar Wavelet	213
11.3	Spectral Analysis of the Stochastic Weights	214
11.3.1	Spectral Analysis of Wavelets	214
11.3.2	Energy Spectrum of the Stochastic Weights	216
11.3.3	Stochastic weights and the Principle of Uncertainty	218
11.4	Conclusions	219
<b>12</b>	<b>Neural Networks Based on the Eigenstates of the Quantum Harmonic Oscillator</b>	221
12.1	Overview	221
12.2	Feed-Forward Neural Networks	222
12.3	Eigenstates of the Quantum Harmonic Oscillator	224
12.4	Neural Networks Based on the QHO Eigenstates	225
12.4.1	The Gauss–Hermite Series Expansion	225
12.4.2	Neural Networks Based on the Eigenstates of the 2D Quantum Harmonic Oscillator	226

12.5	Uncertainty Principles for the QHO-Based Neural Networks .....	228
12.5.1	Uncertainty Principles for Bases and the Balian–Low Theorem.....	228
12.5.2	Uncertainty Principles for Hermite Series.....	229
12.6	Multiscale Modelling of Dynamical System.....	230
12.7	Applications to Image Compression .....	239
12.8	Applications to Fault Diagnosis .....	242
12.8.1	Signals Power Spectrum and the Fourier Transform.....	242
12.8.2	Power Spectrum of the Signal Using the Gauss–Hermite Expansion .....	243
12.8.3	Detection of Changes in the Spectral Content of the System’s Output .....	244
12.9	Conclusions.....	247
<b>13</b>	<b>Quantum Control and Manipulation of Systems and Processes at Molecular Scale .....</b>	<b>251</b>
13.1	Basics of Quantum Systems Control .....	251
13.2	The Spin as a Two-Level Quantum System .....	252
13.2.1	Description of a Particle in Spin Coordinates .....	252
13.2.2	Measurement Operators in the Spin State-Space.....	253
13.2.3	The Spin Eigenstates Define a Two-Level Quantum System .....	254
13.3	The Lindblad and Belavkin Description of Quantum Systems ....	254
13.3.1	The Lindblad Description of Quantum Systems .....	254
13.3.2	The Belavkin Description of Quantum Systems .....	255
13.3.3	Formulation of the Control Problem.....	256
13.4	A Feedback Control Approach for Quantum System Stabilization .....	257
13.4.1	Control Law Calculation Using Schrödinger’s Equation.....	257
13.4.2	Control Law Calculation Using Lindblad’s Equation.....	258
13.5	Simulation Tests .....	260
13.6	Conclusions.....	262
	<b>References.....</b>	<b>263</b>
	<b>Index.....</b>	<b>273</b>