

Contents

Preface — v

Acknowledgements — vi

1 Introduction — 1

1.1 Book content — 2

1.2 Notation — 4

2 Relativistic equations of motion — 9

2.1 Classical equations — 9

2.1.1 Maxwell equations — 9

2.1.2 Equations of motion for a charge in an electromagnetic field — 11

2.1.3 Hamilton–Jacobi equation — 12

2.2 K–G equation — 12

2.2.1 General — 12

2.2.2 Evolution function and completeness relations — 15

2.2.3 Hamiltonian forms of the K–G equation — 16

2.3 Dirac equation — 17

2.3.1 General — 17

2.3.2 Evolution function and completeness relation — 21

2.3.3 Reducing Dirac equation into two independent sets of second-order equations for spinors — 23

2.3.4 Reducing Dirac equation into two independent sets of fourth-order equations for scalar functions — 24

2.3.5 Squaring the Dirac equation — 25

2.4 Spin operators — 29

3 Basic exact solutions — 37

3.1 Free particle motion — 37

3.1.1 Classical motion — 37

3.1.2 States with a given momentum — 38

3.1.3 Positive and negative frequency solutions — 41

3.1.4 Light-cone variables and coherent states — 42

3.1.5 States with given angular momentum projection — 48

3.2 Particles in plane-wave field — 52

3.2.1 Plane-wave electromagnetic field — 52

3.2.2 Classical motion in the plane-wave field — 53

3.2.3 Quantum motion in plane-wave field — 55

3.3 Particles in BGY field — 60

3.3.1	BGY field — 60
3.3.2	Classical motion in a BGY field — 60
3.3.3	Quantum motion in a BGY field — 63
3.4	Particles in a constant and uniform magnetic field — 64
3.4.1	Introduction — 64
3.4.2	Page's and Rabi's solutions — 66
3.4.3	Creation and annihilation operators — 70
3.4.4	Stationary states — 73
3.4.5	Orthonormality and completeness of stationary states — 82
3.4.6	Coherent states — 86
3.4.7	Zero magnetic field limit — 91
3.4.8	Some other types of nonstationary states — 92
3.5	Particles in spherically symmetric fields — 94
3.5.1	General — 94
3.5.2	Separation of variables in K-G and Dirac equations — 97
3.5.3	Specification of potentials and complete classical solution — 99
3.5.4	Azimuthal motion — 102
3.5.5	Radial motion — 107
3.6	Particles in the Aharonov-Bohm field and in its superpositions with other fields — 113
3.6.1	Introduction — 113
3.6.2	Aharonov-Bohm field — 116
3.6.3	Magnetic-solenoid field — 118
3.6.4	Quasicoherent states in the magnetic-solenoid field — 127
3.6.5	Aharonov-Bohm field and additional electromagnetic fields — 134
4	Particles in fields of special structure — 144
4.1	Introduction — 144
4.2	Crossed electromagnetic fields — 145
4.2.1	General — 145
4.2.2	Stationary crossed fields — 148
4.2.3	Nonstationary crossed fields — 152
4.3	Longitudinal electromagnetic fields — 166
4.3.1	General — 166
4.3.2	Longitudinal motion in the electric field — 169
4.3.3	Transversal motion in the magnetic field — 172
4.4	Superposition of crossed and longitudinal fields — 174
4.4.1	General — 174
4.4.2	Crossed and longitudinal electric field — 175
4.4.3	Crossed and longitudinal electric and magnetic fields — 179
4.5	Fields of nonstandard structure — 202

5	Dirac–Pauli equation and its solutions — 214
5.1	Introduction — 214
5.2	Constant and uniform magnetic field — 215
5.3	Plane-wave field — 217
5.4	Superposition of a plane-wave field and a parallel electric field — 220
6	Propagators of relativistic particles — 223
6.1	Introduction — 223
6.2	Proper-time representations for particle propagators — 225
6.2.1	General — 225
6.2.2	Proper-time representations in a constant uniform field and a plane wave field — 228
6.3	Path-integrals for particle propagators — 233
6.3.1	Path integral for K–G propagator — 234
6.3.2	Path integral for the Dirac propagator in even dimensions — 237
6.3.3	Path integral for the Dirac propagator in odd dimensions — 241
6.3.4	Classical and pseudoclassical description of relativistic particles — 243
6.4	Calculations of Dirac propagators using path integrals — 244
6.4.1	Spin factor in $3 + 1$ dimensions — 244
6.4.2	Propagator in the constant uniform electromagnetic field — 248
6.4.3	Propagator in a constant uniform field and a plane wave field — 252
6.4.4	Propagator in a constant uniform field in $2 + 1$ dimensions — 258
7	Electron interacting with a quantized electromagnetic plane wave — 263
7.1	Dirac equation with quantized plane wave — 263
7.1.1	General — 263
7.1.2	Separation of variables — 267
7.2	Quantized monochromatic plane wave with arbitrary polarization — 271
7.3	Quantized plane wave of general form — 273
7.4	Canonical forms for Hamiltonian of quasiphotons — 276
7.5	Stationary and coherent states — 285
7.5.1	Stationary states — 285
7.5.2	Relations of orthogonality, normalization and completeness — 288
7.6	Reduction to Volkov solutions — 290
7.7	Electron interacting with quantized plane-wave and with external electromagnetic background — 292
7.7.1	Classical plane wave along the quantized field — 292
7.7.2	Classical magnetic field directed along the quantized plane wave — 296

7.8	Linear and quadratic combinations of creation and annihilation operators — 299
7.8.1	Linear combinations — 299
7.8.2	Quadratic combinations — 306
8	Spin equation and its solutions — 310
8.1	Introduction — 310
8.2	Associated equations — 312
8.2.1	Associated Schrödinger equations — 312
8.2.2	Dirac-like equation — 312
8.2.3	Rigid rotator equation — 313
8.3	Some properties of the spin equation — 314
8.3.1	The inverse problem — 314
8.3.2	General solution — 315
8.3.3	Stationary solutions — 316
8.3.4	Reduction of the external field — 316
8.3.5	Transformation matrix — 318
8.3.6	Evolution operator — 319
8.4	Self-adjoint spin equation — 320
8.4.1	General solution and inverse problem — 320
8.4.2	Hamiltonian and Lagrangian forms of self-adjoint spin equation — 321
8.5	Exact solutions of spin equation — 322
8.6	Darboux transformation for spin equation — 328
9	One-dimensional Schrödinger equation and its solutions — 332
9.1	ESP I : $V(x) = cx$ — 333
9.2	ESP II : $V(x) = V_1x^2 + V_2x$ — 334
9.3	ESP III : $V(x) = -V_1/x + V_2/x^2$ — 334
9.4	ESP IV : $V(x) = V_1/x^2 + V_2x^2$ — 337
9.5	ESP V : $V(x) = V_1e^{-2cx} + V_2e^{-cx}$ — 338
9.6	ESP VI : $V(x) = V_1/\sin^2 cx + V_2/\cos^2 cx$ — 340
9.7	ESP VII : $V(x) = V_1 \tan^2 cx + V_2 \tan cx$ — 342
9.8	ESP VIII : $V(x) = V_1 \tanh^2 cx + V_2 \tanh cx$ — 343
9.9	ESP IX : $V(x) = V_1 \coth^2 cx + V_2 \coth cx$ — 345
9.10	ESP X : $V(x) = (V_1 + V_2 \cosh 2x)/(\sinh^2 2x)$ — 345
9.11	ESP XI : $V(x) = (V_1 + V_2 \sinh cx)/(\cosh^2 cx)$ — 346
10	Coherent states — 348
10.1	Introduction — 348
10.2	Coherent states of the Heisenberg–Weyl group — 349
10.2.1	HW algebra and HW group — 349
10.2.2	CS of the HW group and Glauber CS — 351

10.2.3	Heisenberg uncertainty relation and CS —	353
10.2.4	Schrödinger–Glauber CS of a harmonic oscillator —	355
10.3	Coherent states for systems with quadratic Hamiltonians —	356
10.3.1	Basic equations —	357
10.3.2	Integrals of motion linear in canonical operators $\hat{q}$ and $\hat{p}$ —	358
10.3.3	Time dependent generalized CS —	359
10.3.4	Standard deviations and uncertainty relations —	361
10.3.5	Simple examples —	362

A Appendix 1 — 367

A.1	Pauli matrices —	367
A.1.1	General properties —	367
A.1.2	Vectors and spinors associated with Pauli matrices —	369
A.1.3	Eigenvalue problem in space of complex spinors —	371
A.1.4	Calculations of matrix elements —	373
A.2	Dirac gamma-matrices —	374
A.2.1	General properties —	374
A.2.2	Gamma-matrix structure of the Lorentz transformation —	377

B Appendix 2 — 380

B.1	Laguerre functions —	380
B.2	Hermite polynomials and Hermite functions —	405

Bibliography — 413

Index — 429