

Contents

Introduction — v

Jeffrey D. Achter and Rachel Pries

Generic Newton polygons for curves of given p -rank — 1

- 1 Introduction — 1
- 2 Structures in positive characteristic — 3
 - 2.1 The p -rank — 3
 - 2.2 Newton polygons — 4
 - 2.3 Semicontinuity and purity — 7
 - 2.4 Notation on stratifications and Newton polygons — 8
- 3 Stratifications on the moduli space of Abelian varieties — 9
 - 3.1 The p -ranks of Abelian varieties — 9
 - 3.2 Newton polygons of Abelian varieties — 10
- 4 The p -rank stratification of the moduli space of stable curves — 11
 - 4.1 The moduli space of stable curves — 11
 - 4.2 The p -rank stratification of $\overline{\mathcal{M}}_g$ — 12
 - 4.3 Connectedness of p -rank strata — 13
 - 4.4 Open questions about the p -rank stratification — 13
- 5 Stratification by Newton polygon — 14
 - 5.1 Newton polygons of curves of small genus — 14
 - 5.2 Generic Newton polygons — 15
- 6 Hyperelliptic curves — 16
- 7 Some conjectures about Newton polygons of curves — 18
 - 7.1 Nonexistence philosophy — 19
 - 7.2 Supersingular curves — 20
 - 7.3 Other nonexistence results — 20

Alp Bassa, Peter Beelen, and Nhut Nguyen

Good towers of function fields — 23

- 1 Introduction — 23
- 2 The Drinfeld modular towers $(X_0(P^n))_{n \geq 0}$ — 25
- 3 An example of a classical modular tower — 32
- 4 A tower obtained from Drinfeld modules over a different ring — 33
 - 4.1 Explicit Drinfeld modules of rank 2 — 33
 - 4.2 Finding an isogeny — 36
 - 4.3 Obtaining a tower — 38

Claude Carlet and Sylvain Guilley

Correlation-immune Boolean functions for easing counter measures to side-channel attacks — 41

- 1 Introduction — 42
- 2 Preliminaries — 45
- 2.1 The combiner model of pseudo-random generator in a stream cipher and correlation-immune functions — 45
- 2.2 Side-channel attacks — 49
- 2.3 Masking counter measure — 51
- 3 Methods for allowing masking to resist higher order side-channel attacks — 53
- 3.1 Leakage squeezing for first-order masking — 53
- 3.2 Leakage squeezing for second-order masking — 55
- 3.3 Rotating S-box masking — 56
- 4 New challenges for correlation-immune Boolean functions — 58
- 4.1 Basic facts on CI functions, orthogonal arrays and dual distance of codes — 58
- 4.2 Known constructions of correlation-immune functions — 61
- 4.3 Synthesis of minimal weights of d -CI Boolean functions — 65

Jung Hee Cheon, Taechan Kim, and Yongsoo Song

The discrete logarithm problem with auxiliary inputs — 71

- 1 Introduction — 72
- 2 Algorithms for the ordinary DLP — 73
- 2.1 Generic algorithms — 73
- 2.2 Nongeneric algorithms — 76
- 3 The DLPwAI and Cheon's algorithm — 78
- 3.1 $p - 1$ cases — 79
- 3.2 Generalized algorithms — 80
- 4 Polynomials with small value sets — 82
- 4.1 Fast multipoint evaluation in a blackbox manner — 82
- 4.2 An approach using polynomials of small value sets — 83
- 5 Approach using the rational polynomials: Embedding to elliptic curves — 84
- 6 Generalized DLPwAI — 85
- 6.1 Representation of a multiplicative subgroup of \mathbb{Z}_{p-1}^\times — 85
- 6.2 A group action on \mathbb{Z}_p^* and polynomial construction — 86
- 6.3 Main result — 86
- 7 Applications and implications — 87
- 7.1 Strong Diffie–Hellman problem and its variants — 87
- 7.2 Attack on the existing schemes using Cheon's algorithm — 88
- 8 Open problems and further work — 89

Massimo Giulietti and Gábor Korchmáros

Garden of curves with many automorphisms — 93

- 1 Introduction — 93
- 2 Notation and background — 94
- 3 Upper bounds on the size of G depending on g — 95
- 4 Upper bounds on the size of the p -subgroups of G depending on the p -rank — 96
- 5 Examples of curves with large automorphism groups — 97
 - 5.1 Curves with unitary automorphism group — 97
 - 5.2 Curves with Suzuki automorphism group — 98
 - 5.3 Curves with Ree automorphism group — 99
 - 5.4 The Giulietti–Korchmáros curve — 99
 - 5.5 The generalized GK curve — 100
 - 5.6 A curve admitting $SU(3, p)$ as an automorphism group — 101
 - 5.7 General hyperelliptic curves with a \mathbb{K} -automorphism 2-group of order $2g + 2$ — 101
 - 5.8 A curve with genus $g = (2^h - 1)^2$ admitting a \mathbb{K} -automorphism 2-group of order of order $2(g - 1) + 2^{h+1} - 2$ — 101
 - 5.9 General bielliptic curves with a dihedral \mathbb{K} -automorphism 2-group of order $4(g - 1)$ — 102
 - 5.10 A curve of genus g with a semidihedral \mathbb{K} -automorphism 2-group of order $2(g - 1)$ — 104
- 6 Characterizations — 105
 - 6.1 Curves with many automorphisms with respect to their genus — 105
 - 6.2 Curves with a large nontame automorphism group — 106
 - 6.3 Theorem 6.2 and some generalizations of Deligne–Lusztig curves — 107
 - 6.4 Group-theoretic characterizations — 109
- 7 The possibilities for G when the p -rank is 0 — 110
- 8 Large automorphism p -groups in positive p -rank — 112
 - 8.1 $p = 2$ — 112
 - 8.2 $p = 3$ — 116
 - 8.3 $p > 3$ — 117

Tor Helleseth

Nonlinear shift registers – A survey and challenges — 121

- 1 Introduction — 121
- 2 Nonlinear shift registers — 123
 - 2.1 The binary de Bruijn graph — 124
 - 2.2 The pure cycling register — 126
 - 2.3 The complementary cycling register — 126
 - 2.4 De Bruijn sequences — 126

3	Mykkeltveit's proof of Golomb's conjecture — 129
4	The D-morphism — 132
5	Conjugate pairs in PCR — 134
6	Finite fields and conjugate pairs — 135
6.1	Cycle joining and cyclotomy — 137
7	Periodic structure of NLFSRs — 139
8	Conclusions — 142

Florian Pausinger and Alev Topuzoğlu

Permutations of finite fields and uniform distribution modulo 1 — 145

1	Introduction — 145
2	Preliminaries — 146
3	Good and weak families of permutations — 150
4	Existence of good families — 151
5	Permutation polynomials of Carlitz rank 3 — 152
6	Bounds for $f(S_p^\sigma)$ — 154
7	Computational results — 156
8	Concluding remarks — 157

Alexander Pott, Kai-Uwe Schmidt, and Yue Zhou

**Semifields, relative difference sets,
and bent functions — 161**

1	Introduction — 161
2	Semifields — 162
3	Relative difference sets — 165
4	Relative difference sets and semifields — 167
5	Planar functions in odd characteristic — 171
6	Planar functions in characteristic 2 — 172
7	Component functions of planar functions — 173
8	Concluding remarks and open problems — 175

Ron Steinfeld

**NTRU cryptosystem: Recent developments and emerging mathematical problems in
finite polynomial rings — 179**

1	Introduction — 179
2	Notation and preliminaries — 181
2.1	Notation — 181
2.2	Probability and algorithms — 181
2.3	Rings — 182
2.4	Lattices — 182

3	Review of the NTRU cryptosystem —	183
3.1	The NTRU construction —	183
3.2	Security of NTRU: Computational/statistical problems and known attacks —	185
4	Recent developments in security analysis of NTRU —	189
4.1	Overview —	189
4.2	Gaussian distributions modulo lattices and Fourier analysis —	192
4.3	Statistical hardness of the NTRU decision key cracking problem —	195
4.4	Computational hardness of the ciphertext cracking problem —	198
5	Recent developments in applications of NTRU —	200
5.1	NTRU-based homomorphic encryption —	200
5.2	NTRU-based multilinear maps —	204
6	Conclusions —	207

Gabriel D. Villa-Salvador

Analog of the Kronecker–Weber theorem in positive characteristic — 213

1	Introduction —	213
2	The classical case —	215
3	A proof of the Kronecker–Weber theorem based on ramification groups —	216
4	Cyclotomic function fields —	219
5	The maximal Abelian extension of k —	221
6	Reciprocity law —	223
7	The proof of David Hayes —	224
8	Witt vectors and the conductor —	225
8.1	The conductor —	228
8.2	The conductor according to Schmid —	228
9	The Kronecker–Weber–Hayes theorem —	229
10	Final remarks —	235

Index — 239