
Contents

Preface	VII
1 Topology of Digital Images: Basic Ingredients	1
1.1 What Is a Digital Image?	2
1.2 Digital Visual Space	4
1.3 Creating Your Own Images	6
1.4 Randomly Generated Images	9
1.5 Ways to Display Images	10
1.6 Digital Image Formats	11
1.7 Image Data Types	12
1.8 Colour Images	15
1.8.1 Colour Spaces	15
1.8.2 Colour Channels	16
1.9 Colour Lookup Table	18
1.10 Accessing Values of Pixels	20
1.11 RGB, Greyscale, and Binary (BW) Images	21
1.12 Displaying Colour Channels	22
1.13 Metric Space	25
1.14 Neighbourhood of a Point	25
1.15 Set Interior, Set Boundary and Set Complement	27
1.16 Descriptive Neighbourhoods of Picture Points	29
1.16.1 Unbounded Descriptive Neighbourhoods	31
1.16.2 Bounded Descriptive Neighbourhoods	32
1.16.3 Indistinguishable Bounded Descriptive Neighbourhoods	33
1.16.4 Descriptive Neighbourhood Patterns	33
1.17 Open Sets and Closed Sets	34
1.17.1 Set Interior and Open Neighbourhoods Revisited	37
1.18 Spatially Near Sets	39
1.18.1 Closure of a Set	42
1.19 Descriptively Near Sets	43
1.19.1 Descriptively Near, Spatially Far	45
1.20 Continuity	47
1.20.1 Proximally Continuous	48
1.20.2 Adherent Points	48

1.20.3	Smirnov Proximal Continuity	49
1.20.4	Significance of Shape-Based Adherence in Generating Set Patterns	51
1.21	Dense Subsets and Taimanov's Theorem	53
1.21.1	Dense Subsets in Pictures	53
1.21.2	Descriptive Closure of a Set	53
1.21.3	Descriptively Dense Subsets in Pictures	54
1.21.4	Fenestation Yields Dense Subsets	56
1.21.5	Dense Subspaces and Taimanov's Theorem	57
1.22	Pattern Similarity Distance and Pattern Saliency	59
1.22.1	First and Second Countable Picture Spaces	61
1.23	Nearness Collections	62
1.24	Digital Image Structures	63
1.24.1	Topological Structures in Digital Images	64
1.24.2	Visual Sets and Metric Topology	65
1.24.3	Descriptive Nearness, Descriptive Remoteness	65
1.25	Problems	68
2	Structures Arising from Sets of Pixels	77
2.1	Picture Elements	79
2.2	Logical Operations on Images	83
2.2.1	Not (Inverting Image Pixel Values)	83
2.2.2	XOR Operation on Pairs of Binary Images	84
2.3	Separating Image Foreground from Background	87
2.4	Conjunction of Thresholded Colour Channels	88
2.5	Improving Contrast in an Image	90
2.6	Gamma Transform	92
2.7	Distance Functional and Closure of a Set	95
2.8	Metric Proximities	98
2.9	Proximal Neighbourhood	100
3	Visualisations and Covers	103
3.1	Histograms and Plots	104
3.1.1	Colour Histograms	109
3.2	Adaptive Thresholding	109
3.3	Contrast Stretching	112
3.4	Histogram Matching	114
3.5	Open Covers and Compact Subsets	116
3.6	Lindelöf spaces and Characterisations of Compactness	117
3.7	How to Topologize Everything	119
4	Linear Filtering and Visual Patterns in Images	121
4.1	Set Patterns	123
4.1.1	Descriptive Point Set Pattern	123
4.1.2	Picture Set Patterns	124

4.1.3	Motif Set Patterns	125
4.1.4	Descriptive Motif Set Patterns	127
4.2	Two Local Set Pattern Generation Methods	130
4.2.1	Spatial Proximity Pattern Generation Method	130
4.2.2	Descriptive Proximity Pattern Generation Method...	132
4.3	Cluster-Based Patterns	133
4.4	Filter Kernels	134
4.5	Linear Filter Experiments	135
4.6	Linear Convolution Filtering	136
4.7	Selecting a Region-of-Interest	137
4.8	Adding Noise to Image	139
4.9	Mean Filtering	140
4.10	Median Filtering	141
4.11	Rank Order Filtering	142
4.12	Normal Distribution Filtering	144
4.13	Commonest Filter	144
4.14	Set Structures in Digital Images	145
4.14.1	Gap between Points and Sets	146
4.14.2	Sufficient Nearness of Points and Sets	147
4.14.3	Sufficient Nearness of Sets	148
4.15	Descriptive EF-Proximity Space	151
4.15.1	Near Sets in L-Proximity Spaces	152
4.15.2	Descriptive L-Proximity	155
4.16	Uniform Topologies on an Image	159
4.16.1	Cluster of Sets and Point Clusters	162
4.16.2	Forgery Test: Application of Descriptive Point Clusters	164
4.16.3	Interior of a Set	166
4.16.4	Sufficient Nearness Closure of a Set	167
4.17	Neighbourhood of a Set and Neighbourhood Filters	169
5	Edges, Lines, Ridges, and Nearness Structures	173
5.1	Edge Detection	176
5.2	Enhancing Digital Image Edges	177
5.3	Laplace of Gaussian Filter Image Enhancement	179
5.4	Zero-Cross Edge Filter Image Enhancement	180
5.5	Metric Nearness and Collar Sets in Digital Images	182
5.6	Herrlich Nearness Structures	183
5.6.1	Descriptive Nearness Structures	184
5.7	Nearness of Collections	186
5.7.1	Spatially Near Collections	186
5.7.2	Descriptively Near Collections	189
5.8	Hyperspaces in Digital Images	194

6	Corners and Symmetric Proximity	199
6.1	Anisotropy vs. Isotropy in Edge Detection	199
6.2	Detecting Edges and Lines in Digital Images	202
6.3	Detecting Image Corners	203
6.4	Nearness of Sets of Pixels	204
6.5	Proximal Neighbourhood	209
7	Separation of Image Regions and Set Patterns	211
7.0.1	Proximal Framework and Proximal Framology	214
7.0.2	Topology of Digital Images	216
7.1	Image Dilation	216
7.2	MM Erosion	220
7.3	Opening and Closing Operations	220
7.4	Watershed Segmentation Method	221
7.5	Point Detection	223
7.6	Discovery of the Separation Axioms	225
7.6.1	R0 Symmetric Space	226
7.6.2	T0 Anti-symmetric Space	226
7.6.3	T1 Space	228
7.6.4	Hausdorff T2 Space	231
7.6.5	T3 Space	234
7.6.6	T4 Space	235
7.7	Visual Set Patterns in Descriptive Separation Spaces	236
7.7.1	Visual Patterns in Descriptive T1 Spaces	236
7.7.2	Visual Patterns in Descriptive T2 Spaces	239
7.7.3	Set Pattern Generators	241
8	Descriptive Raster Spaces	247
8.1	Introduction	247
8.2	Separation of Raster Image Regions	249
8.3	2D Image from a Color Image	251
8.4	Median Filtered Vector to Raster Image	252
8.5	Hausdorff Raster Spaces	253
8.6	Uniform Covering	258
8.7	Vector Field Visualization	258
8.8	Constructing 3D Scenes with Projective Transformations ...	261
9	Component Analysis and Uniform Spaces	265
9.1	Principal Component Analysis	266
9.2	Entourage Uniformity	272
9.3	Entourage Uniformity Set Patterns	274
9.4	Covering Uniformity	276

10	Shapes and Shape Set Patterns	279
10.1	Overview of Shape Descriptors	281
10.2	Form Shape Descriptor	290
10.3	Zernike Moments	292
10.4	Hu Spatial Moments	296
10.5	Shape Set Patterns	297
11	Texture and Texture Set Patterns	301
11.1	Texture Features	303
11.2	Lowpass Image Filtering with Local Texture Features	308
11.3	Experimenting with Statistical Measures of Texture	309
11.4	Texture Set Patterns	311
11.4.1	Proximity in Compact Hausdorff Spaces	311
11.4.2	Set Patterns Arising from Comparable Image Textures	314
12	Pattern-Based Picture Classification	317
12.1	Pattern-Based Classification	318
12.1.1	Classification Example	319
12.2	Sample Results of Experiments	324
12.3	Pattern Generators	325
12.3.1	Feature Space for Descriptive Patterns	330
12.3.2	Pattern-Based Nearness Measures	330
12.4	Stability in Pattern Constructions	337
12.4.1	Multiple Pattern Generation Stability	339
12.4.2	Comparison with Existing Clustering Stability Analysis	340
A	Appendix: Matlab and Mathematica Scripts	343
A.1	Matlab Scripts from Ch. 1	343
A.2	Matlab Scripts from Ch. 2	351
A.3	Matlab and Mathematica9 Scripts from Ch. 3	354
A.4	Matlab and Mathematica9 Scripts from Ch. 4	357
A.5	Matlab and Mathematica9 Scripts from Ch. 5	361
A.6	Matlab and Mathematica9 Scripts from Ch. 6	361
A.7	Matlab and Mathematica9 Scripts from Ch. 7	363
A.8	Matlab Scripts from Ch. 8	364
A.9	Matlab and Mathematica Scripts from Ch. 9	366
A.10	Matlab and Mathematica Scripts from Ch. 10	367
A.11	Matlab and Mathematica Scripts from Ch. 11	370
B	Notes and Further Readings	373
	References	383
	Author Index	399
	Subject Index	403