

Contents

1 Preliminaries	1
1.1 One-Dimensional Markov Processes	1
1.2 Brownian Motion and Diffusion on $\mathbb{R}$	2
1.3 Stochastic Integrals and Itô's Formula	7
1.4 Poisson Process	9
1.5 Modified Bessel Functions	11
1.6 Generalised Functions and Integral Transforms	13
References	18
2 Telegraph Process on the Line	19
2.1 Definition of Process and the Structure of Distribution	20
2.2 Kolmogorov Equations	22
2.3 Telegraph Equation	27
2.4 Characteristic Function	30
2.5 Transition Density	32
2.6 Convergence to Brownian Motion	36
2.7 Laplace Transforms	40
References	42
3 Functionals of Telegraph Process	45
3.1 Motions with Barriers	45
3.1.1 Telegraph Process with Reflecting Barrier	45
3.1.2 Telegraph Process with Absorbing Barrier	47
3.2 Occupation Time Distributions	51
3.2.1 Feynmann-Kac Connection	52
3.2.2 Statement of the Main Result	55
3.2.3 Proof of Theorems 3.6 and 3.7	57
3.3 First Passage Time	62
References	67

4	Asymmetric Jump-Telegraph Processes	69
4.1	Generalised Jump-Telegraph Processes	69
4.1.1	Transition Densities	70
4.1.2	Expectations and Variances Jump-Telegraph Martingales	75
4.1.3	Change of Measure for Jump-Telegraph Processes	79
4.2	Moments	81
	References	88
5	Financial Modelling and Option Pricing	89
5.1	Hedging Strategies and Option Pricing	90
5.1.1	Option Pricing, Hedging and Martingales	90
5.1.2	Black-Scholes Model and Girsanov's Theorem	92
5.2	Market Model Based on Jump-Telegraph Processes	94
5.3	Diffusion Rescaling and Natural Volatility	96
5.4	Fundamental Equation and Perfect Hedging	103
5.5	Pricing Call Options	105
5.6	Historical and Implied Volatilities in the Jump Telegraph Model	116
5.6.1	Historical Volatility	116
5.6.2	Implied Volatility and Numerical Results	118
5.7	Pricing Exotic Options	121
	References	124
	Index	127