

Contents

1	Random Walks, a Good Place to Begin	1
1.1	Nearest Neighbor Random Walks on \mathbb{Z}	1
1.1.1	Distribution at Time n	2
1.1.2	Passage Times via the Reflection Principle	3
1.1.3	Some Related Computations	4
1.1.4	Time of First Return	7
1.1.5	Passage Times via Functional Equations	8
1.2	Recurrence Properties of Random Walks	9
1.2.1	Random Walks on \mathbb{Z}^d	9
1.2.2	An Elementary Recurrence Criterion	10
1.2.3	Recurrence of Symmetric Random Walk in \mathbb{Z}^2	12
1.2.4	Transience in \mathbb{Z}^3	14
1.3	Exercises	17
2	Doebelin's Theory for Markov Chains	25
2.1	Some Generalities	25
2.1.1	Existence of Markov Chains	26
2.1.2	Transition Probabilities & Probability Vectors	27
2.1.3	Transition Probabilities and Functions	28
2.1.4	The Markov Property	30
2.2	Doebelin's Theory	30
2.2.1	Doebelin's Basic Theorem	30
2.2.2	A Couple of Extensions	33
2.3	Elements of Ergodic Theory	35
2.3.1	The Mean Ergodic Theorem	36
2.3.2	Return Times	37
2.3.3	Identification of π	41
2.4	Exercises	43
3	Stationary Probabilities	49
3.1	Classification of States	49
3.1.1	Classification, Recurrence, and Transience	50

3.1.2	Criteria for Recurrence and Transience	52
3.1.3	Periodicity	56
3.2	Computation of Stationary Probabilities	58
3.2.1	Preliminary Results	58
3.2.2	Computations via Linear Algebra	59
3.3	Wilson's Algorithm and Kirchhoff's Formula	64
3.3.1	Spanning Trees and Wilson Runs	64
3.3.2	Wilson's Algorithm	65
3.3.3	Kirchhoff's Matrix Tree Theorem	68
3.4	Exercises	69
4	More About the Ergodic Properties of Markov Chains	73
4.1	Ergodic Theory Without Doeblin	74
4.1.1	Convergence of Matrices	74
4.1.2	Abel Convergence	75
4.1.3	Structure of Stationary Distributions	78
4.1.4	A Digression About Moments of Return Times	80
4.1.5	A Small Improvement	82
4.1.6	The Mean Ergodic Theorem Again	84
4.1.7	A Refinement in the Aperiodic Case	85
4.1.8	Periodic Structure	90
4.2	Exercises	91
5	Markov Processes in Continuous Time	99
5.1	Poisson Processes	99
5.1.1	The Simple Poisson Process	99
5.1.2	Compound Poisson Processes on \mathbb{Z}^d	102
5.2	Markov Processes with Bounded Rates	104
5.2.1	Basic Construction	105
5.2.2	An Alternative Construction	108
5.2.3	Distribution of Jumps and Jump Times	111
5.2.4	Kolmogorov's Forward and Backward Equations	112
5.3	Unbounded Rates	114
5.3.1	Explosion	114
5.3.2	Criteria for Non-explosion or Explosion	120
5.3.3	What to Do when Explosion Occurs	122
5.4	Ergodic Properties	122
5.4.1	Classification of States	123
5.4.2	Stationary Measures and Limit Theorems	126
5.4.3	Interpreting and Computing $\hat{\pi}_{ii}$	129
5.5	Exercises	130
6	Reversible Markov Processes	137
6.1	Reversible Markov Chains	138
6.1.1	Reversibility from Invariance	138
6.1.2	Measurements in Quadratic Mean	139

6.1.3	The Spectral Gap	141
6.1.4	Reversibility and Periodicity	143
6.1.5	Relation to Convergence in Variation	144
6.2	Dirichlet Forms and Estimation of β	145
6.2.1	The Dirichlet Form and Poincaré's Inequality	146
6.2.2	Estimating β_+	148
6.2.3	Estimating β_-	150
6.3	Reversible Markov Processes in Continuous Time	151
6.3.1	Criterion for Reversibility	151
6.3.2	Convergence in $L^2(\hat{\pi})$ for Bounded Rates	152
6.3.3	$L^2(\hat{\pi})$ -Convergence Rate in General	154
6.3.4	Estimating λ	157
6.4	Gibbs States and Glauber Dynamics	157
6.4.1	Formulation	158
6.4.2	The Dirichlet Form	159
6.5	Simulated Annealing	162
6.5.1	The Algorithm	163
6.5.2	Construction of the Transition Probabilities	164
6.5.3	Description of the Markov Process	166
6.5.4	Choosing a Cooling Schedule	166
6.5.5	Small Improvements	169
6.6	Exercises	170
7	A Minimal Introduction to Measure Theory	179
7.1	A Description of Lebesgue's Measure Theory	179
7.1.1	Measure Spaces	179
7.1.2	Some Consequences of Countable Additivity	181
7.1.3	Generating σ -Algebras	182
7.1.4	Measurable Functions	183
7.1.5	Lebesgue Integration	184
7.1.6	Stability Properties of Lebesgue Integration	186
7.1.7	Lebesgue Integration on Countable Spaces	188
7.1.8	Fubini's Theorem	190
7.2	Modeling Probability	192
7.2.1	Modeling Infinitely Many Tosses of a Fair Coin	193
7.3	Independent Random Variables	194
7.3.1	Existence of Lots of Independent Random Variables	194
7.4	Conditional Probabilities and Expectations	196
7.4.1	Conditioning with Respect to Random Variables	198
	References	199
	Index	201