

# Contents

<b>1 Prologue: What This Book Is About</b>	<b>1</b>
1.1 Part I: The Basic Model	1
1.1.1 Adaptive vs. Non-adaptive Search	2
1.1.2 $Q$ -ary Search with Lies	3
1.1.3 Half-Lies, Erasures and Other Types of Errors	4
1.1.4 Heuristics	4
1.2 Part II: More Models and Applications	4
1.2.1 Erasure Errors, Delays and Time-outs	5
1.2.2 Group Testing	5
1.2.3 Memory Faults and Resilient Search	5
1.2.4 A Model of Learning	6
1.3 Bibliographic Notes	6
 <b>Part I The Ulam-Rényi Game and Its Variants</b>	
<b>2 Fault-Tolerant Search à la Ulam-Rényi</b>	<b>9</b>
2.1 Introduction	9
2.1.1 The Binary Ulam-Rényi Game	10
2.2 The Volume Bound	12
2.3 Borderline States Satisfying the Volume Bound with Equality	17
2.4 The Solution of the 20 Question Game with Lies	21
2.5 Asymptotics for the Ulam-Rényi Problem	23
2.6 Heuristics for the Ulam-Rényi Problem	24
2.6.1 Experimental Validation of the Heuristics	27
2.7 Bibliographic Notes	28
2.8 Exercises	29
<b>3 Adaptive vs. Non-adaptive Search</b>	<b>31</b>
3.1 Coding in a Channel with Noiseless Feedback	31
3.2 No Feedback Equals Error-Correcting Codes	32

3.3	Elements of the Theory of Error-Correcting Codes .....	33
3.3.1	Linear and Hamming Codes.....	35
3.3.2	MDS Codes and the Reed-Solomon Codes.....	36
3.3.3	Bounds on Codes .....	38
3.4	Fault-Tolerant $q$ -ary Search and Minimum Feedback.....	40
3.4.1	Fault-Tolerant $q$ -ary search.....	42
3.4.2	Perfect Strategies and Least Adaptiveness: $M = q^m$ .....	44
3.4.3	Arbitrary Cardinality of the Search Space: Least Adaptive Quasi-perfect Strategies .....	51
3.5	Some Finite Exact Results for the $q$ -ary Adaptive Ulam-Rényi game .....	56
3.6	Bibliographic Notes .....	62
3.7	Exercises.....	63
<b>4</b>	<b>Weighted Errors over a General Channel</b> .....	<b>65</b>
4.1	Introduction .....	65
4.2	Two-Batch Search with Weighted Lies .....	65
4.3	The Lower Bound: The Winning Strategy .....	68
4.3.1	The First Batch of Questions .....	68
4.3.2	The Second Batch of Questions .....	70
4.4	The Upper Bound .....	71
4.5	Other Noise Models: Unidirectional Errors.....	74
4.6	Bibliographic Notes .....	75
4.7	Exercises.....	76
<b>5</b>	<b>Variations on a Theme of Ulam and Rényi: More Types of Questions and Lies</b> .....	<b>77</b>
5.1	Comparison-Based Search: The Multiple-Interval Queries .....	77
5.2	Query Optimal Multi-interval Search with $k = O(e^2)$ .....	78
5.2.1	The Case of Two Lies: A Canonical Representation of States and 2-Interval Queries .....	87
5.2.2	About Comparison Questions .....	89
5.3	Linearly Bounded Number of Lies .....	90
5.4	Prefix-Bounded Numbers of Lies .....	96
5.5	Bibliographic Notes .....	97
5.6	Exercises.....	97
 <b>Part II Other Models</b>		
<b>6</b>	<b>Delays and Time Outs</b> .....	<b>101</b>
6.1	Search with Fixed Batches of Questions and Variable Delay in Answers .....	102
6.2	Search with Variable Batches and Delays.....	115
6.3	Lost Answers and Delays.....	122
6.3.1	Extensions and Generalizations .....	127
6.3.2	The Proof of Proposition 6.1 .....	129

6.3.3	Broadcast with Latency vs. Search with Delay .....	133
6.4	Bibliographic Notes .....	136
6.5	Exercises .....	137
<b>7</b>	<b>Group Testing</b> .....	139
7.1	Group Testing with Subset Tests .....	140
7.1.1	The $(p, v, n)$ -SUPER-SELECTOR .....	141
7.1.2	Approximate Group Testing .....	143
7.1.3	Bounds on the Size of a $(p, v, n)$ -SUPER-SELECTOR .....	144
7.2	Interval Group Testing .....	147
7.2.1	Non-adaptive Fault-Tolerant Interval Group Testing .....	149
7.2.2	Two-Stage Fault-Tolerant Interval Group Testing .....	151
7.3	Some Typical Applications of Group Testing in Computational Biology .....	169
7.4	Bibliographic Notes .....	171
7.5	Exercises .....	172
<b>8</b>	<b>Resilient Search</b> .....	175
8.1	The Definition of the Problem and a Lower Bound .....	175
8.2	Randomized Resilient Search .....	177
8.3	Optimal Deterministic Resilient Search .....	179
8.3.1	The Verification Procedure .....	181
8.4	Bibliographic Notes .....	183
8.5	Exercises .....	184
<b>9</b>	<b>A Model for Learning</b> .....	185
9.1	Computational Learning .....	185
9.2	Predicting from Expert Advice .....	186
9.3	Learning in Noisy Environments .....	188
9.3.1	Rényi's Probabilistic Example .....	188
9.3.2	Learning with Negative Reinforcement .....	188
9.3.3	Supervised Learning for Online Prediction .....	192
9.4	Bibliographic Notes .....	197
9.5	Exercises .....	198
	<b>Bibliography</b> .....	199