

Contents

Part I Three Physics Visions

1	Notes on “Quantum Gravity” and Noncommutative Geometry	3
	J.M. Gracia-Bondía	
1.1	Introduction	3
1.2	Gravity and Experiment: Expect the Unexpected	6
1.2.1	Noncommutative Geometry I	10
1.2.2	Whereto Diffeomorphism Invariance?	10
1.3	Gravity from Gauge Invariance in Field Theory	13
1.3.1	Preliminary Remarks	14
1.3.2	Exempli Gratiae	15
1.3.3	The Free Lagrangian	16
1.3.4	A Canonical Setting	17
1.3.5	What to Expect	19
1.3.6	Causal Gauge Invariance by Brute Force	21
1.3.7	CGI at All Orders: Going for It	25
1.3.8	Details on Quantization and Graviton Helicities	27
1.3.9	Final Remarks	30
1.3.10	Other Ways	31
1.4	The Unimodular Theories	31
1.5	The Noncommutative Connection	34
1.5.1	Prolegomena	34
1.5.2	Ironies of History	35
1.5.3	Spectral Triples	36
1.5.4	On the Reconstruction Theorem	40
1.5.5	The Noncommutative Torus	41
1.5.6	The Noncompact Case	43
1.5.7	Nc Toric Manifolds (Compact and Noncompact)	45
1.5.8	Closing Points	46
1.5.9	Some Interfaces with Quantum Gravity	51
1.6	More on the “Cosmological Constant Problem” and the Astroparticle Interface	51
	References	54

2 Quantum Gravity as Sum over Spacetimes	59
J. Ambjørn, J. Jurkiewicz and R. Loll	
2.1 Introduction	59
2.2 CDT	66
2.3 Numerical Results	74
2.3.1 The Emergent de Sitter Background	75
2.3.2 Fluctuations Around de Sitter Space	79
2.3.3 The Size of the Universe and the Flow of G	83
2.4 Two-Dimensional Euclidean Quantum Gravity	87
2.4.1 Continuum Formulation	87
2.4.2 The Lattice Regularization	89
2.4.3 Counting Graphs	93
2.4.4 The Continuum Limit	98
2.5 Two-Dimensional Lorentzian Quantum Gravity	102
2.6 Matrix Model Representation	109
2.6.1 The Loop Equations	112
2.6.2 Summation over All Genera in the CDT Matrix Model	116
2.7 Discussion and Perspectives	119
References	122
 3 Lectures on Quantization of Gauge Systems	 125
N. Reshetikhin	
3.1 Introduction	125
3.2 Local Lagrangian Classical Field Theory	127
3.2.1 Space – Time Categories	127
3.2.2 Local Lagrangian Classical Field Theory	128
3.2.3 Classical Mechanics	128
3.2.4 First-Order Classical Mechanics	129
3.2.5 Scalar Fields	130
3.2.6 Pure Euclidean d -Dimensional Yang–Mills	131
3.2.7 Yang–Mills Field Theory with Matter	131
3.2.8 Three-Dimensional Chern–Simons Theory	132
3.3 Hamiltonian Local Classical Field Theory	133
3.3.1 The Framework	133
3.3.2 Hamiltonian Formulation of Local Lagrangian Field Theory	134
3.4 Quantum Field Theory Framework	137
3.4.1 General Framework of Quantum Field Theory	137
3.4.2 Constructions of Quantum Field Theory	138
3.5 Feynman Diagrams	141
3.5.1 Formal Asymptotic of Oscillatory Integrals	141
3.5.2 Integrals Over Grassmann Algebras	146
3.5.3 Formal Asymptotics of Oscillatory Integrals Over Super-manifolds	149
3.5.4 Charged Fermions	152

3.6	Finite-Dimensional Faddeev–Popov Quantization and the BRST Differential	153
3.6.1	Faddeev–Popov Trick	153
3.6.2	Feynman Diagrams with Ghost Fermions	156
3.6.3	Gauge Independence	158
3.6.4	Feynman Diagrams for Linear Constraints	159
3.6.5	The BRST Differential	161
3.7	Semiclassical Quantization of a Scalar Bose Field	163
3.7.1	Formal Semiclassical Quantum Mechanics	163
3.7.2	$d > 1$ and Ultraviolet Divergencies	167
3.8	The Yang–Mills Theory	169
3.8.1	The Gauge Fixing	170
3.8.2	The Faddeev–Popov Action and Feynman Diagrams	170
3.8.3	The Renormalization	171
3.9	The Chern–Simons Theory	173
3.9.1	The Gauge Fixing	174
3.9.2	The Faddeev–Popov Action in the Chern–Simons Theory	174
3.9.3	Vacuum Feynman Diagrams and Invariants of 3-Manifolds	177
3.9.4	Wilson Loops and Invariants of Knots	182
3.9.5	Comparison with Combinatorial Invariants	186
	References	188

Part II Novel Mathematical Tools

4	Mathematical Tools for Calculation of the Effective Action in Quantum Gravity	193
	I.G. Avramidi	
4.1	Introduction	193
4.2	Effective Action in Quantum Field Theory and Quantum Gravity	195
4.2.1	Non-gauge Field Theories	197
4.2.2	Gauge Field Theories	203
4.2.3	Quantum General Relativity	205
4.3	Heat Kernel Method	208
4.3.1	Laplace-Type Operators	209
4.3.2	Spectral Functions	212
4.3.3	Heat Kernel	213
4.3.4	Asymptotic Expansion of the Heat Kernel	214
4.3.5	Zeta-Function and Determinant	221
4.4	Green Function	223
4.5	Heat Kernel Coefficients	227
4.5.1	Non-recursive Solution of the Recursion Relations	227
4.5.2	Covariant Taylor Basis	229
4.5.3	Matrix Algorithm	230
4.5.4	Diagrammatic Technique	232

4.5.5	General Structure of Heat Kernel Coefficients	234
4.6	High-Energy Approximation	236
4.7	Low-Energy Approximation	238
4.7.1	Algebraic Approach	239
4.7.2	Covariantly Constant Background in Flat Space	241
4.7.3	Homogeneous Bundles over Symmetric Spaces	245
4.8	Low-Energy Effective Action in Quantum General Relativity	255
	References	258
5	Lectures on Cohomology, T-Duality, and Generalized Geometry	261
	P. Bouwknegt	
5.1	Cohomology and Differential Characters	261
5.1.1	A Brief Review of de Rham and Čech Cohomology	262
5.1.2	Electromagnetism	264
5.1.3	The Čech – de Rham Complex	267
5.1.4	Differential Cohomologies	270
5.2	T-Duality	276
5.2.1	Introduction to T-Duality	276
5.2.2	The Buscher Rules	277
5.2.3	Gysin Sequences and Dimensional Reduction	282
5.2.4	T-Duality = Takai Duality	285
5.2.5	T-Duality as a Duality of Loop Group Bundles	287
5.3	Generalized Geometry	290
5.3.1	Cartan Relations	291
5.3.2	Lie Algebroids	292
5.3.3	Generalized Geometry	293
5.3.4	Courant Algebroids	297
5.3.5	Generalized Complex Geometry	303
5.3.6	T-Duality in Generalized Geometry	307
	References	309
6	Stochastic Geometry and Quantum Gravity: Some Rigorous Results	313
	H. Zessin	
6.1	An Axiomatic Introduction into Regge's Model	314
6.2	The Zero-Infinity Law of Stochastic Geometry and the Cluster Process	316
6.2.1	Basic Concepts	316
6.2.2	Cluster Properties and the Zero-Infinity Law of Stochastic Geometry	317
6.2.3	Cluster Properties and the Zero-Infinity Law for Marked Configurations	319
6.2.4	The Cluster Process	320
6.2.5	The Poisson Process P_ρ	322
6.3	Poisson–Delaunay Surfaces with Intrinsic Random Metric	323
6.3.1	The Delaunay Cluster Property	323

6.3.2	Poisson–Delaunay Surfaces	326
6.3.3	Scholion: The Voronoi Cluster Property	327
6.4	Ergodic Behaviour of $\mathcal{PD}$ -Surfaces	328
6.4.1	Palm Measures and Palm Distributions	328
6.4.2	An Ergodic Theorem for Intrinsic Metric Quantities of Stationary Random Surfaces	329
6.4.3	Curvature Properties of the Poisson–Delaunay Surface	331
6.5	The Two-Dimensional Regge Model of Pure Quantum Gravity	333
6.6	Comments and Final Dreams	334
	References	335

Part III Afterthoughts

7	Steps Towards Quantum Gravity and the Practice of Science: Will the Merger of Mathematics and Physics Work?	339
	B. Booß-Bavnbek	
7.1	Regarding the Need and the Chances of Unification	339
7.2	The Place of Physics in John Dee’s <i>Groundplat of Sciences and Artes, Mathematicall</i> of 1570	340
7.3	Delimitation Between Mathematics and Physics	341
7.4	Variety of Modelling Purposes	342
7.4.1	Production of Data, Model-Based Measurements	342
7.4.2	Simulation	343
7.4.3	Prediction	343
7.4.4	Control	344
7.4.5	Explain Phenomena	345
7.4.6	Theory Development	346
7.5	“The Trouble with Physics”	347
7.6	Theory–Model–Experiment	347
7.6.1	First Principles	348
7.6.2	Towards a Taxonomy of Models	348
7.6.3	The Scientific Status of Quantum Gravity as Compared to Medicine and Economics	350
7.7	General Trends of Mathematization and Modelling	351
7.7.1	Deep Divide	351
7.7.2	Charles Sanders Peirce’s Semiotic View	351
	References	353
	Index	355