Contents

1	Galois Theory				
	1.1	Action of a Solvable Group and Representability by Radicals 2	2		
		1.1.1 A Sufficient Condition for Solvability by Radicals	3		
		1.1.2 The Permutation Group of the Variables and Equations			
		of Degree 2 to 4			
		1.1.3 Lagrange Polynomials and Commutative Matrix Groups . 5	5		
		1.1.4 Solving Equations of Degree 2 to 4 by Radicals 8	3		
	1.2	Fixed Points under an Action of a Finite Group and Its Subgroups			
	1.3	Field Automorphisms and Relations Between Elements in a Field 14	1		
		1.3.1 Separable Equations	1		
		1.3.2 Algebraicity over the Invariant Subfield	5		
		1.3.3 Subalgebra Containing the Coefficients of the Lagrange			
		Polynomial	ó		
		1.3.4 Representability of One Element Through Another			
		Element over the Invariant Field	7		
	1.4	Action of a <i>k</i> -Solvable Group and Representability by <i>k</i> -Radicals	3		
	1.5	Galois Equations)		
	1.6	Automorphisms Connected with a Galois Equation			
	1.7	The Fundamental Theorem of Galois Theory	2		
		1.7.1 Galois Extensions	2		
		1.7.2 Galois Groups	3		
		1.7.3 The Fundamental Theorem	1		
		1.7.4 Properties of the Galois Correspondence	1		
		1.7.5 Change of the Coefficient Field	5		
	1.8	A Criterion for Solvability of Equations by Radicals	7		
		1.8.1 Roots of Unity	7		
		1.8.2 The Equation $x^n = a$	3		
		1.8.3 Solvability by Radicals)		
	1.9	A Criterion for Solvability of Equations by <i>k</i> -Radicals 30)		
		1.9.1 Properties of k -Solvable Groups)		

viii Contents

		1.9.2 1.9.3	Solvability by k -Radicals	32
			in k -Radicals	33
	1.10	Unsol	vability of Complicated Equations by Solving Simpler	
		Equati	ions	35
			A Necessary Condition for Solvability	35
			Classes of Finite Groups	36
	1.11	Finite	Fields	37
2	Cov	erings		41
	2.1	Cover	ings over Topological Spaces	42
		2.1.1	Coverings and Covering Homotopy	42
		2.1.2	Classification of Coverings with Marked Points	43
		2.1.3	Coverings with Marked Points and Subgroups	
			of the Fundamental Group	45
		2.1.4	Coverings and Galois Theory	48
	2.2	Comp	letion of Finite Coverings over Punctured Riemann Surfaces	52
		2.2.1	Filling Holes and Puiseux Expansions	52
		2.2.2	Analytic-Type Maps and the Real Operation of Filling	
			Holes	54
		2.2.3	Finite Ramified Coverings with a Fixed Ramification Set	57
		2.2.4	Riemann Surface of an Algebraic Equation over the Field	
			of Meromorphic Functions	62
3	Ran	nified C	Coverings and Galois Theory	65
	3.1		Ramified Coverings and Algebraic Extensions of Fields	00
			romorphic Functions	66
		3.1.1	The Field $P_a(O)$ of Germs at the Point $a \in X$	
			of Algebraic Functions with Ramification over O	66
		3.1.2	Galois Theory for the Action of the Fundamental Group	
			on the Field $P_a(O)$	68
		3.1.3	Field of Functions on a Ramified Covering	71
	3.2	Geom	etry of Galois Theory for Extensions of a Field	
			romorphic Functions	72
		3.2.1	Galois Extensions of the Field $K(X)$	72
		3.2.2	Algebraic Extensions of the Field of Germs	
			of Meromorphic Functions	73
		3.2.3	Algebraic Extensions of the Field of Rational Functions.	74
Ref	erenc	es		79
	1			
ınd	ex .			81