## **Contents**

1	Intro	duction		1	
	1.1	The Simulation Pipeline			
	1.2	Introdu	action to Modeling	4	
		1.2.1	General Prerequisites	5	
		1.2.2	Derivation of Models	7	
		1.2.3	Analysis of Models	9	
		1.2.4	Classification of Models	11	
		1.2.5	Scales	11	
	action to Simulation	13			
		1.3.1	General Remarks	13	
		1.3.2	Assessment	14	
2	Requ	Required Tools in Short			
	2.1	Eleme	ntary and Discrete Topics	18	
	2.2	Contin	uous Aspects	19	
		2.2.1	Linear Algebra	19	
		2.2.2	Analysis	21	
		2.2.3	Significance for Modeling and Simulation	28	
	2.3				
		2.3.1	Why Randomness?	29	
		2.3.2	Discrete Probability Spaces	30	
		2.3.3	Continuous Probability Spaces	36	
		2.3.4	Asymptotics	41	
		2.3.5	Statistical Inference	43	
	2.4	Numer	rical Aspects	47	
		2.4.1	Basics	47	
		2.4.2	Interpolation and Quadrature	51	
		2.4.3	Direct Solution of Linear Systems of Equations	59	
		2.4.4	Iteration Methods	61	

х

		<ul><li>2.4.5 Ordinary Differential Equations</li><li>2.4.6 Partial Differential Equations</li></ul>	68 78		
	2.5	Interrelationships Between Tools and Applications	83		
Par	t I	Gaming—Deciding—Planning: A Warm-up for Modeling			
3	Ga	me Theory	87		
	3.1	Games in Strategic Normal Form	88		
	3.2	Games Without Assumptions on the Opponent	90		
	3.3	1 11 0	91		
	3.4	Dominant Strategies	93		
	3.5	Nash Equilibria	94		
	. 3.6	Mixed Strategies	95		
	3.7	Outlook	97		
4	Group Decision Making				
•	4.1	Individual Preferences and Group Decision Making	100		
	4.2	• •	103		
	4.3		106		
5	Sol	heduling	111		
3	5.1				
	5.2		112 118		
	5.2	<u> </u>	124		
	5.3 5.4	•	128		
	-	· ·			
6		iener Processes	131		
	6.1	L.	132		
	6.2	¥ 1	134		
	6.3		135		
	6.4	Application: Development of Money Investments	138		
Par	.4 TT	Troffe on Highways and Data Highways, Once			
rai	111	Traffic on Highways and Data Highways: Once Through the Simulation Pipeline			
		Through the Simulation Pipeline			
7	Ma	acroscopic Simulation of Road Traffic	149		
	7.1		150		
	7.2	2 Homogeneous Traffic Flow	152		
		7.2.1 An Initial Result	152		
		7.2.2 Velocity, Flow and Density	153		
		7.2.3 Fundamental Diagram	154		
		7.2.4 Model Refinements	155		
	7.3	Inhomogeneous Traffic Flow	158		
	7.4	Simulation of a Simple Circular Road	160		
		7.4.1 A First Attempt	161		
		7.4.2 An Improved Simulation	163		

	7.5 7.6	Signal and Traffic VelocitySummary and Outlook			
8		·	171		
Ü	8.1		172		
	0.1		172		
			. <i>,                                   </i>		
	8.2	311.2 Italia Italia	176		
	8.3		178 178		
	0.5		179 179		
			179 180		
		,	180 182		
	0.4				
	8.4	<b>6</b> · · · · ·	186		
			186		
			188		
			193		
	8.5		199		
	8.6	Summary and Outlook	201		
9	Stoch	nastic Traffic Simulation	203		
	9.1	Model Approach	204		
	9.2	Queuing Systems	206		
		9.2.1 Stochastic Processes	207		
		9.2.2 Classification of Elementary Queuing Systems 2	213		
		9.2.3 Examples for the Kendall Notation	214		
		9.2.4 Performance Indices and First Results	215		
	9.3	Queuing Networks	218		
			219		
			220		
	9.4		222		
			223		
			229		
			234		
		ξ ξ	235		
	9.5		237		
		•			
Par	t III	Dynamical Systems: Cause, Effect, and Interplay			
10	Popu	lation Dynamics	241		
-	10.1	•	242		
	10.2		242		
	<b></b>		243		
			243		
	10.3	<u> </u>	245		
	10.4		250		

xii Contents

11	Conti	ol Engineering	255	
	11.1	The Basics of Control Theory	256	
		11.1.1 Control Loop	257	
		11.1.2 Description of Linear Dynamical Systems	258	
		11.1.3 Requirements for the Controller	258	
		11.1.4 PID Controller	259	
	11.2	Exemplary Modeling of a Multibody System	261	
		11.2.1 Linearized Model with Conservation		
		of Linear and Angular Momentum	263	
		11.2.2 Complete Model with Lagrange Equations	266	
			270	
	11.3	Fuzzy Set Theory	271	
		11.3.1 Membership in Fuzzy Sets	271	
			274	
			276	
			277	
	11.4		280	
		11.4.1 Fuzzification	281	
			282	
		11.4.3 Defuzzification	283	
		11.4.4 Example	284	
	11.5	Fuzzy Control of the Inverted Pendulum	284	
		11.5.1 Parameters and Constraints	285	
		11.5.2 Swinging Up the Pendulum	286	
		11.5.3 Stabilizing the Pendulum	288	
	11.6	Outlook	288	
12	Chao	s Theory	291	
	12.1 Introduction			
	12.2	From Order to Chaos	293	
		12.2.1 Logistic Mapping and Its Fixed Points	293	
		12.2.2 Numerical Analysis and Bifurcations	295	
		12.2.3 Transition into Chaos	298	
	12.3	Strange Attractors	301	
		12.3.1 Self-Similarity and Fractal Dimension	302	
		12.3.2 Hénon Mapping	304	
		12.3.3 General Two-Dimensional Quadratic Mapping	305	
	12.4	Chaotic Behavior of a Driven Pendulum	307	
		12.4.1 Model of the Pendulum	308	
		12.4.2 Discretization	309	
		12.4.3 Cycles and Attractors	310	

Contents xiii

Part IV	Physics in the Computer: Take-Off Toward Number
	Crunching

13	Mote	•	namics	317
	13.1	Modeli	ng of Molecules and Interactions	318
		13.1.1	Fundamental Physical Forces	318
		13.1.2	Potentials for Uncharged Atoms	319
		13.1.3	Computation of the Force Acting on an Atom	323
	13.2	Equation	ons of Motion and Their Solutions	324
		13.2.1	Equations of Motion	324
		13.2.2	Euler's Method	325
		13.2.3	Velocity-Störmer-Verlet	326
		13.2.4	Remarks	327
	13.3	Simula	tion Domain	328
		13.3.1	NVT Ensemble	328
		13.3.2	Boundary Conditions	329
	13.4	Implem	nentation	330
		13.4.1	Linked-cells Data Structure	331
	13.5	Parallel	lization	333
	13.6	Outloo	k	335
11	Hank	Tuesde	_	337
14	14.1		rion of the Heat Equation	338
	14.1	14.1.1	Number of Dimensions	340
	14.2		ization	341
	14.2	14.2.1	3-Point-Stencil	342
		14.2.1	5-Point-Stencil	344
		14.2.2	Boundary Treatment	346
	142		ical Solution of the PDE	346
	14.3	14.3.1	Simple Relaxation Methods	347
		14.3.1	Multigrid Methods	348
		14.3.2	Wullight Wethous	340
15	Fluid	l Dynam	ics	355
	15.1	Fluids	and Flows	356
	15.2	Mather	natical Model	357
		15.2.1	Navier-Stokes Equations	357
		15.2.2	Remarks Concerning the Derivation	360
	15.3	Discret	ization of the Navier–Stokes Equations	361
		15.3.1	Finite Differences	361
		15.3.2	Treatment of Spatial Derivatives	362
		15.3.3	Treatment of Temporal Derivatives	363
		15.3.4	Treatment of Boundary Conditions	364
	15.4	Numer	ical Solution of the Discretized Equations	365
		15.4.1	Time Step	365
		15.4.2	Spatially Discrete Momentum Equations	367

xiv Contents

		15.4.3	Spatially Discrete Poisson Equation for the Pressure	. 367
		15.4.4	Regarding Stability	. 368
	15.5	Applica	ation Example: Flow Around an Obstacle	. 368
	15.6	Outlool	k	. 370
		15.6.1	Problem Settings and Models	. 370
		15.6.2	Discretizations	. 371
		15.6.3	Structured Grids	. 373
		15.6.4	Unstructured Grids	. 376
		15.6.5	Approaches for the Treatment of Changing	
			Geometries	. 379
16	Glob	al Illumi	ination in Computer Graphics	. 381
	16.1	Quantit	ties from Radiometry	. 382
	16.2	The Rendering Equation	ndering Equation	. 384
	16.3	Technic	ques for the Solution of the Rendering Equation	. 387
		16.3.1	Ray Tracing	. 388
		16.3.2	Path-Tracing	. 390
		16.3.3	Further Ray-Tracing Derivates	. 391
	16.4	The Ra	diosity Method	. 392
		16.4.1	Basic Principle	. 392
		16.4.2	Computation of the Form Factors	. 394
		16.4.3	Solution of the Radiosity Equation	. 396
		16.4.4	Remarks and Improvements	. 397
Co	ncludii	ng Rema	rks	. 401
Re	ference	es		. 403
Inc	lex			40