Contents

1	A Brief Historical Introduction and Motivations	1
2	Surfaces with Constant Mean Curvature 2.1 The Variational Formula for the Area 2.2 The Hopf Differential 2.3 Elliptic Equations for cmc Surfaces	13 13 24 26
3	The Comparison and Tangency Principles	37 37 44 49
4	Constant Mean Curvature Embedded Surfaces 4.1 The Alexandrov Reflection Method	55 56 68 72
5	The Flux Formula for Constant Mean Curvature Surfaces 5.1 The Flux Formula	81 89 92 97
6	The Area and the Volume of a Constant Mean Curvature Surface . 6.1 A Monotonicity Formula for the Area 6.2 Consequences of the Height Estimate 6.3 Surfaces with Small Volume 6.4 Surfaces with Large Volume	103 103 108 110 115
7	Constant Mean Curvature Disks with Circular Boundary 7.1 Disks with Small Area 7.2 Stable cmc Disks 7.3 Other Area Estimates of a cmc Disk	123 123 127 131

xiii

xiv	Content

8	The Dirichlet Problem of the cmc Equation	133
	8.1 The Method of Continuity	134
	8.2 The Bounded Slope Condition	142
	8.3 Graphs on Convex Domains	145
	8.4 The Exterior Circle Condition	156
	8.5 Radial Graphs in Euclidean Space	162
9	The Dirichlet Problem in Unbounded Domains	173
	9.1 Height Estimates	173
	9.2 Graphs on Convex Domains	179
	9.3 The Exterior Circle Condition	182
10	Constant Mean Curvature Surfaces in Hyperbolic Space	189
	10.1 Basics on Hyperbolic Geometry	190
	10.2 The Flux Formula	197
	10.3 Embedded cmc Surfaces in \mathbb{H}^3	200
	10.4 Graphs in Hyperbolic Space	205
	10.5 The Case $ H \le 1$	211
	10.6 Surfaces with Circular Boundary	214
11	The Dirichlet Problem in Hyperbolic Space	219
	11.1 Preliminaries and Definitions	219
	11.2 Graphs on a Horosphere	223
	11.3 Graphs on a Geodesic Plane and an Equidistant Surface	228
12	Constant Mean Curvature Spacelike Surfaces in Lorentz-	
	Minkowski Space	235
	12.1 Basics on Lorentz-Minkowski Space	236
	12.2 The Mean Curvature of a Spacelike Surface	240
	12.3 Spacelike Surfaces with Non-empty Boundary	243
	12.4 The Flux Formula	246
	12.5 A Monotonicity Formula for the Area	251
	12.6 The Dirichlet Problem	253
App	pendix A The Variation Formula for the Area and for the Volume .	261
Ap	pendix B Open Questions	269
Ref	erences	273
Ind	ex	289