

Short Contents

Preface	xxi
Acknowledgments	xxiii
1 How to Use This Book	1
1.1 Getting Started	1
1.2 Overview	2
1.3 Instructions for Using Specific Programs	2
2 Fundamentals of Acoustics	5
2.1 Basic Physical Principles and Units	5
2.2 Vibration and Waves	8
2.3 Sound and Hearing	30
3 Signals and Systems	45
3.1 Analog Signals and the Fourier Transform	45
3.2 Digital Signals, DFT, FFT	63
3.3 Systems and Filters	78
3.4 Dynamic Systems and Feedback Control	109
4 Computer Programs and Programming Languages	163
4.1 Csound	163
4.2 Max	176
4.3 Mathematica	179
4.4 C / C++	190
4.5 Processing	203
5 Fundamentals of Sound Synthesis	207
5.1 Fundamental Techniques of Sound Synthesis	207
5.2 Additive Synthesis	244
5.3 Subtractive Synthesis	259
6 Nonlinear Techniques	267
6.1 Modulation Techniques and Distortion	267
6.2 Nonlinear Systems	297
7 Other Techniques for Sound Analysis and Synthesis	317
7.1 Granular Synthesis	317
7.2 Special Analysis Methods	328
8 Physical Modeling	337
8.1 Mass-Spring Models	338
8.2 Wave Guides	390

9	Sound and Space	417
9.1	Spatial Hearing.	417
9.2	Reflection and Reverberation	426
9.3	Sound Reproduction	435
10	Computers and Composition	455
10.1	Chance and Probability	455
10.2	Stochastic Processes	487
10.3	Other Techniques Used for Composition	508
Appendix A	Fundamentals of Mathematics	545
Appendix B	Tables	571
Bibliography		577
Index		587

Contents

Preface to the Original German Edition	xxi
Preface to the English Edition	xxii
Acknowledgements	xxiii
1 How to Use This Book	1
1.1 Getting Started	1
1.2 Overview	2
1.3 Instructions for Using Specific Programs	2
1.3.1 Using the Mathematica Notebooks	3
1.3.2 Using the Csound Programs	3
1.3.3 Using the C/C++ Programs	3
1.3.4 Using the Max Patches	4
1.3.5 Using the Processing Programs	4
2 Fundamentals of Acoustics	5
2.1 Basic Physical Principles and Units	5
2.1.1 Path, Velocity, Acceleration	5
2.1.2 Mass and Force	6
2.1.3 Momentum, Work, Power, Energy	7
2.2 Vibration and Waves	8
2.2.1 Harmonic Oscillation	8
2.2.1.1 Definition and Mathematical Representation	8
2.2.1.2 Damped Oscillation	10
2.2.1.3 The Addition of Harmonic Oscillations	11
2.2.1.4 Beats	13
2.2.1.5 Natural Vibrations	13
2.2.1.6 Driven Oscillation and Resonance	15
2.2.2 Periodic Vibrations and their Spectrum	16
2.2.2.1 The Definition of Periodic Vibrations	16
2.2.2.2 Standard Examples	16
2.2.2.3 Other Examples	16
2.2.2.4 Constructing Periodic Oscillation from Harmonic Waveforms	17
2.2.2.5 The Spectrum of Periodic Oscillations	18
2.2.3 Aperiodic Oscillation	20
2.2.3.1 Non-harmonic Partial	20
2.2.3.2 Noise	20
2.2.3.3 Pulses	21
2.2.3.4 Quasi-periodic Oscillation	22
2.2.3.5 Variable Frequencies	22
2.2.4 Waves	23
2.2.4.1 Definition and Examples	23
2.2.4.2 Mathematical Description	24

	2.2.4.3	Superposition of Waves	25
	2.2.4.4	Propagation of Waves	27
	2.2.4.5	The Doppler Effect	29
2.3	Sound and Hearing		30
2.3.1	Pitch		30
	2.3.1.1	Frequency Range and Octaves	30
	2.3.1.2	The Harmonic Series and Pure Intervals	31
	2.3.1.3	Intervals	32
2.3.2	Timbre		33
	2.3.2.1	Periodic Vibration	33
	2.3.2.2	Formants	33
	2.3.2.3	Spectra of Natural Sounds	34
	2.3.2.4	Non-Harmonic Spectra	35
	2.3.2.5	Fusion	36
	2.3.2.6	Missing Fundamentals and Residue Pitch	36
2.3.3	Loudness		37
	2.3.3.1	Sound Power and Sound Intensity	37
	2.3.3.2	Decibels	37
	2.3.3.3	Phons	38
2.3.4	Interaction of the Parameters		38
	2.3.4.1	Frequency Groups	39
	2.3.4.2	Auditory Masking	39
	2.3.4.3	Combination Tones	40
	2.3.4.4	Particular Aspects of Pitch Perception	41
2.3.5	Resonant Spaces		41
	2.3.5.1	Reflections and Reverberation	42
	2.3.5.2	Sound Localization	43
	2.3.5.3	Distance Perception	44
3 Signals and Systems			45
3.1	Analog Signals and the Fourier Transform		45
3.1.1	Periodic Signals and Fourier Series		46
	3.1.1.1	Fourier Series	46
	3.1.1.2	Calculating the Coefficients	46
	3.1.1.3	Examples	48
3.1.2	Complex Representation		50
	3.1.2.1	Complex Numbers	50
	3.1.2.2	Trigonometric Representation	51
	3.1.2.3	The Exponential Form	52
	3.1.2.4	Algebra With Complex Numbers	53
	3.1.2.5	Rotating Complex Vectors	55
	3.1.2.6	Fourier Analysis in Complex Representation	58
3.1.3	Aperiodic Signals and Fourier Integrals		60
	3.1.3.1	Definition	60
	3.1.3.2	Examples	61

3.1.4	Analog Systems	62
3.1.4.1	Definition and Examples	62
3.1.4.2	Linear Differential Equations	62
3.1.4.3	Laplace Transform ($\rightarrow$ CD)	62
3.2	Digital Signals, DFT, FFT	63
3.2.1	Digital Representation of Signals	63
3.2.1.1	Sampling	63
3.2.1.2	Representations and Simple Examples	64
3.2.1.3	Characteristics and Basic Transformations	64
3.2.1.4	Quantization	66
3.2.1.5	Aliasing	66
3.2.1.6	Rotating Vectors	67
3.2.2	The Discrete Fourier Transform DFT	68
3.2.2.1	Calculating the Coefficients of Discrete Fourier Series	68
3.2.2.2	Examples	68
3.2.2.3	Signals With Unknown or Non-Integer Periods	69
3.2.2.4	The Complex Form and the Characteristics of the DFT	70
3.2.2.5	The Fast Fourier Transform FFT	73
3.2.3	The Z-Transform	76
3.2.3.1	Definition and Transformation of Simple Signals	76
3.2.3.2	Characteristics and Examples	77
3.3	Systems and Filters	78
3.3.1	Systems	78
3.3.1.1	Definition and Examples	78
3.3.1.2	General Properties of Systems	79
3.3.1.3	Impulse Response and Convolution	79
3.3.1.4	Properties of Systems in the Frequency Domain	80
3.3.1.5	The Complex Representation	82
3.3.1.6	Filters	84
3.3.1.7	The Transfer Function and the Z-Plane	85
3.3.1.8	Linear Phase Filters	89
3.3.1.9	Filter Design	90
3.3.2	Non-Recursive Filters / FIR Filters	91
3.3.2.1	Definitions and Properties	91
3.3.2.2	Implementation	91
3.3.2.3	Designing Filters by Setting Zeros	93
3.3.2.4	Fourier Approximation	94
3.3.2.5	Windowing	95
3.3.3	Recursive Filters / IIR Filters	96
3.3.3.1	Definition and Properties	96
3.3.3.2	Implementation	96
3.3.3.3	Designing Filters by Setting Poles and Zeros	99
3.3.3.4	Stability	100
3.3.3.5	Special Filters	100
3.3.3.6	Filter Design by Transforming Analog into Digital Systems	108

3.4	Dynamic Systems and Feedback Control.	109
3.4.1	Differential Equations	110
3.4.1.1	Introductory Example, Phase Space	110
3.4.1.2	The Types of Differential Equations and Further Definitions	111
3.4.1.3	Stationary System Analysis, Characteristic Curves	114
3.4.1.4	Numerical Methods, Difference Equations	116
3.4.1.5	The Pendulum	119
3.4.2	Fixed Points and Attractors	122
3.4.2.1	Fixed Points	122
3.4.2.2	Catastrophes	124
3.4.2.3	Attractors.	127
3.4.3	Chaos	128
3.4.3.1	Introductory Example	128
3.4.3.2	Conditions for Chaos	129
3.4.3.3	Chaos in Time-Discrete Systems	130
3.4.3.4	Chaos in Differential Equations	133
3.4.3.5	Multiple Solutions and Discontinuities	135
3.4.4	Techniques of Feedback Control.	137
3.4.4.1	Introductory Concepts and Examples	137
3.4.4.2	Elements of Control Circuits	139
3.4.4.3	Feedback Circuits	144
3.4.4.4	Nonlinear Feedback Circuits	145
3.4.4.5	The Computation of Feedback Control	148
3.4.4.6	Control By Filters	149
3.4.4.7	Examples	150
3.4.5	Synchronization	153
3.4.5.1	Self-Sustained Oscillators	154
3.4.5.2	The Van der Pol Oscillator	154
3.4.5.3	Synchronization Using Periodic Excitation	156
3.4.5.4	Mutual Synchronization of Weakly Coupled Oscillators	158
3.4.5.5	Synchronization of Chaotic Oscillators	160
4	Computer Programs and Programming Languages	163
4.1	Csound	163
4.1.1	The Syntax of the Csound Orchestra	163
4.1.1.1	The Structure of the Orchestra	163
4.1.1.2	Constants and Variables	164
4.1.1.3	Functions	165
4.1.1.4	Assignment, Operators, Expressions and Conditional Expressions	165
4.1.1.5	Control Flow	166
4.1.1.6	Simple Signal Generators ($\rightarrow$ CD).	167
4.1.1.7	Signal Modifiers ($\rightarrow$ CD)	167
4.1.1.8	Delay Lines	167
4.1.1.9	Sound Input and Output	167
4.1.1.10	Opcodes	168

4.1.2	The Score	168
4.1.3	Function Table Generators	169
4.1.4	Generating Scores and Events	171
4.1.4.1	Generating Events With Csound Instruments	171
4.1.4.2	Generating Scores With Programs in C	172
4.1.4.3	Generating Scores and Events Using <i>Processing</i>	173
4.1.4.4	Generating Scores and Events With <i>Max</i>	175
4.1.4.5	Generating Scores With <i>Python</i>	175
4.2	Max	176
4.2.1	Fundamentals	176
4.2.2	Feedback.	176
4.2.3	mxj-Objects	177
4.3	Mathematica	179
4.3.1	Fundamentals	179
4.3.2	Sounds	183
4.3.3	Graphics	185
4.4	C / C++	190
4.4.1	Fundamentals	190
4.4.1.1	The Structure of a C Program	190
4.4.1.2	Types	191
4.4.1.3	Derived and Composite Types	191
4.4.1.4	Operators, Expressions, Mathematical Functions	193
4.4.1.5	Control Flow	194
4.4.1.6	Functions	195
4.4.1.7	Input and Output	197
4.4.1.8	Classes and Objects (→ CD)	199
4.4.1.9	Reading and Writing Binary Data	199
4.4.2	Generating and Storing Sounds	200
4.4.2.1	Raw Data	200
4.4.2.2	Sound Files	201
4.5	Processing	203
4.5.1	Fundamentals	203
4.5.2	Simulations	205
4.5.3	Libraries	206

5 Fundamentals of Sound Synthesis 207

5.1	Fundamental Techniques of Sound Synthesis	207
5.1.1	Overview	207
5.1.1.1	Instruments and Their Schematic Diagrams	207
5.1.1.2	Techniques for Sound Synthesis	208
5.1.1.3	Programs and Programming Languages	209
5.1.1.4	Tables	209
5.1.1.5	Audio Signals and Control Signals.	210
5.1.1.6	Interpolation	210
5.1.1.7	Program Control and Conditional Statements	211

5.1.2	Unit Generators	211
5.1.2.1	The Oscillator	211
5.1.2.2	The Pulse Generator	214
5.1.2.3	The Noise Generator	215
5.1.3	Control Signals	217
5.1.3.1	Simple Control Functions	217
5.1.3.2	Splines	219
5.1.3.3	Interpolation Filters	223
5.1.3.4	Variable Control Signals	224
5.1.3.5	Tempo Functions	226
5.1.3.6	Synchronization	235
5.1.4	Delay Lines	237
5.1.4.1	Definition and Direct Implementation	237
5.1.4.2	The Circular Buffer	237
5.1.4.3	Delay Lines With Variable Delay	238
5.1.4.4	Delay Lines With Feedback	242
5.1.4.5	Applications	244
5.2	Additive Synthesis	244
5.2.1	The Synthesis of Periodic Waveforms	244
5.2.1.1	Basic Techniques	244
5.2.1.2	Variable Parameters	245
5.2.1.3	Fusion	246
5.2.1.4	Data Reduction	247
5.2.1.5	Acoustic Illusions	247
5.2.2	Analysis-Resynthesis	249
5.2.2.1	Introduction	250
5.2.2.2	The Discrete Fourier Transform DFT	251
5.2.2.3	Long-Term Fourier Transform LTFT	254
5.2.2.4	Short-Term Fourier Transform STFT	256
5.2.2.5	The Phase Vocoder	258
5.3	Subtractive Synthesis	259
5.3.1	Filters	259
5.3.1.1	Characteristics of Filters in the Frequency Domain	259
5.3.1.2	Types of Filters	260
5.3.1.3	Special Filters	261
5.3.1.4	Combining Filters	261
5.3.1.5	Effects in the Time Domain	262
5.3.1.6	Variable Filters	263
5.3.2	Applications	263
5.3.2.1	Sound Sources	263
5.3.2.2	Resonators and Formants	264
5.3.2.3	Linear Prediction.	266
6	Nonlinear Techniques	267
6.1	Modulation Techniques and Distortion	267

6.1.1	Amplitude Modulation and Ring Modulation	267
6.1.1.1	Introductory Example	267
6.1.1.2	Basic Techniques	268
6.1.1.3	The Spectrum of Amplitude Modulated Waveforms	269
6.1.2.4	Ring Modulation	271
6.1.2	Frequency Modulation and Phase Modulation	272
6.1.2.1	Introductory Example	272
6.1.2.2	The Basic Method	272
6.1.2.3	The Spectrum of a Frequency Modulated Waveform.	274
6.1.2.4	The Proportion $fc : fm$	278
6.1.2.5	Variable Spectra	278
6.1.2.6	Synthesis Models and Examples	279
6.1.2.7	Extensions of the Basic Method	280
6.1.2.8	The Influence of the Phase	284
6.1.3	Nonlinear Distortion – Waveshaping	285
6.1.3.1	Introductory Examples	286
6.1.3.2	Waveshaping	288
6.1.3.3	The Modulation Index	289
6.1.3.4	Polynomials as Transfer Functions	290
6.1.3.5	Chebyshev Polynomials as Transfer Functions	292
6.1.3.6	Limiters, Compressors and Expanders	294
6.2	Nonlinear Systems	297
6.2.1	Non-Recursive Systems With a Single Input	297
6.2.1.1	Functions of a Single Input Value	297
6.2.1.2	Functions of More Than One Input Value	297
6.2.2	Non-Recursive Systems With More Than One Input Signal	302
6.2.2.1	Functions Using a Single Value of Each Input	302
6.2.2.2	Special Functions of Two Input Signals	303
6.2.2.3	Functions of Several Values of Several Input Signals	307
6.2.3	Recursive Systems	308
6.2.3.1	Functions of One Value	308
6.2.3.2	Functions of Two Values	310
6.2.4	Time-Variant Systems	311
6.2.4.1	Delimiting Systems	311
6.2.4.2	Non-Recursive Systems With Constant Delay	311
6.2.4.3	Non-Recursive Systems With Variable Delay	312
6.2.4.4	Recursive Systems With Constant Delay	313
6.2.4.5	Recursive Systems With Variable Delay	315
7	Other Techniques for Sound Analysis and Synthesis	317
7.1	Granular Synthesis	317
7.1.1	Fundamentals	317
7.1.1.1	Grains	317
7.1.1.2	Techniques for Making and Controlling Grains	319
7.1.1.3	Synchronous Granular Synthesis	321

7.1.1.4	Asynchronous Granular Synthesis	322
7.1.2	Applications	323
7.1.2.1	FOF	323
7.1.2.2	VOSIM	324
7.1.2.3	Granulating Sampled Sounds	326
7.2	Special Analysis Methods	328
7.2.1	Walsh Synthesis	328
7.2.1.1	Walsh Functions	329
7.2.1.2	Examples	329
7.2.2	The Logarithmic Frequency Range in Spectral Analysis	330
7.2.3	Wavelets	331
7.2.3.1	Wavelets	331
7.2.3.2	The Continuous Wavelet Transform	332
7.2.3.3	The Discrete Wavelet Transform	335
8	Physical Modeling	337
8.1	Mass-Spring Models	338
8.1.1	Systems With One Mass	338
8.1.1.1	Harmonic Oscillation	338
8.1.1.2	Exciting the Oscillation	342
8.1.1.3	Damped Harmonic Oscillation	342
8.1.1.4	Exciting the Damped Oscillation	345
8.1.1.5	Oscillation With Nonlinear Acceleration	347
8.1.1.6	Calculations	348
8.1.2	Systems With Two Masses.	349
8.1.2.1	The Oscillation of Two Coupled Masses	349
8.1.2.2	Excitation and Damping	351
8.1.2.3	Nonlinear Acceleration	352
8.1.2.4	Computing the Frequencies of the Natural Resonances	353
8.1.2.5	Calculations (→ CD)	355
8.1.3	The Linear Arrangement of Coupled Masses.	355
8.1.3.1	A Model With Three Masses.	355
8.1.3.2	The String	356
8.1.3.3	Correcting the Dispersion Relation.	358
8.1.3.4	Damping and Nonlinearity	360
8.1.3.5	Picking Up the Sound	361
8.1.3.6	Exciting the String	362
8.1.3.7	Harmonics	363
8.1.4	Two-Dimensional Arrangements of Coupled Masses	365
8.1.4.1	An Example With Three Masses	365
8.1.4.2	Representing the Plane by a Regular Grid	366
8.1.4.3	Resonant Frequencies of the Grid	367
8.1.4.4	Objects With Curved Edges	369
8.1.4.5	Objects With Freely Oscillating Edges	369
8.1.4.6	Rigid Body Motion	370

8.1.4.7	Retroflex Surfaces	371
8.1.4.8	A Grid With Unequal Distances Between the Masses	372
8.1.4.9	Irregular Grids	374
8.1.4.10	Irregular Density or Elasticity	376
8.1.5	Three-Dimensional Arrangements of Coupled Masses	377
8.1.5.1	Longitudinal and Transversal Oscillations of a Mass	377
8.1.5.2	Subdividing Space With a Regular Grid	378
8.1.5.3	Bodies in Free Motion	380
8.1.5.4	A Model With Fixed Points	383
8.1.5.5	Variable Fixed Points	385
8.1.5.6	The Effect of Gravity	386
8.1.5.7	Damping	386
8.1.6	Arbitrary Configurations and Variations	387
8.1.6.1	Coupled Strings	387
8.1.6.2	Geometrically Impossible Shapes	388
8.1.6.3	Spaces of More Than Three Dimensions	389
8.2	Wave Guides	390
8.2.1	Simple Delay Lines	390
8.2.1.1	Delay Lines	390
8.2.1.2	Simple Damping	392
8.2.1.3	Frequency	393
8.2.1.4	Nonlinearity	396
8.2.1.5	The Excitation	397
8.2.1.6	The Karplus-Strong Algorithm	398
8.2.2	Waveguides	399
8.2.2.1	The Ideal Waveguide	400
8.2.2.2	Reflection	401
8.2.2.3	The Advancing Wave Front as Solution of the Wave Equation	402
8.2.2.4	Other Variables for Representing Waves	404
8.2.3	Sound Pickup and Excitation	405
8.2.3.1	The Postions of Pickup and Excitation	405
8.2.3.2	The Duration of the Excitation	407
8.2.3.3	Excitation Without Feedback	408
8.2.3.4	Excitation With Feedback	410
8.2.3.5	Selective Reflection and Sound Radiation	413
8.2.3.6	Harmonics	414
9	Sound and Space	417
9.1	Spatial Hearing	417
9.1.1	Sound Localization	418
9.1.1.1	Interaural Time Difference ITD	418
9.1.1.2	Interaural Intensity Difference IID	419
9.1.1.3	Head-Related Transfer Function HRTF	421
9.1.2	Distance	422
9.1.2.1	The Decrease of Sound Intensity With Distance	422

9.1.2.2	Proportion of Indirect Sound	423
9.1.3	Movement of Sound in Space	424
9.1.3.1	The Doppler Effect	424
9.1.3.2	Additional Information Through Position Change	425
9.2	Reflection and Reverberation	426
9.2.1	Reflections	426
9.2.1.1	Geometrical Considerations	426
9.2.1.2	Scatter and Absorbtion	430
9.2.2	Reverberation	430
9.2.2.1	The Nature of Reverberation	431
9.2.2.2	Simple Reverberators	431
9.2.2.3	Frequency Dependency	434
9.2.2.4	More Complex Filters	435
9.2.2.5	Convolution With an Impulse Response	435
9.3	Sound Reproduction	435
9.3.1	Ideal Solutions	435
9.3.1.1	Simulation of the Sound Source	436
9.3.1.2	Sound Field Reproduction	436
9.3.1.3	The Sound Wave in the Auditory Canal	437
9.3.2	Practical Solutions	437
9.3.2.1	Stereo	438
9.3.2.2	Ambisonics	438
9.3.2.3	Decorrelation	451
10	Computers and Composition	455
10.1	Chance and Probability	455
10.1.1	Fundamentals of Combinatorics	455
10.1.1.1	Introductory Examples	455
10.1.1.2	Permutations	456
10.1.1.3	Combinations	456
10.1.1.4	Arrangements	457
10.1.1.5	Ordering Permutations	458
10.1.1.6	Binomial and Polynomial Coefficients	458
10.1.2	Fundamentals of Probability Calculus	458
10.1.2.1	Standard Examples and Definitions	458
10.1.2.2	Combining Events	459
10.1.2.3	The Terminology and Axioms of Probability Theory.	459
10.1.2.4	Conditional Probability and Stochastic Independence	461
10.1.2.5	Examples	461
10.1.3	Probability, Density and Distribution Functions of Random Variables	461
10.1.3.1	Random Variables and the Probability Function	462
10.1.3.2	Continuous Random Variables and Their Density Function	462
10.1.3.3	Distribution Functions	463
10.1.3.4	Continuous Density Functions and Their Distributions	464
10.1.3.5	Functions of Random Variables	465

10.1.3.6	Measuring Probability Distribution	465
10.1.3.7	Parametric Control of Functions	467
10.1.4	Generating Random Numbers With a Given Density or Distribution.	467
10.1.4.1	Pseudorandom Numbers	467
10.1.4.2	Direct Methods for Generating a Given Distribution	469
10.1.4.3	Inverting the Distribution Function	470
10.1.4.4	Rejection Sampling	472
10.1.4.5	Tables of Elements With Specified Frequency of Occurrence	473
10.1.5	Particular Distributions	476
10.1.5.1	Continuous Uniform Distribution	476
10.1.5.2	Trapezoid Distribution	476
10.1.5.3	Binomial Distribution	477
10.1.5.4	Poisson Distribution	478
10.1.5.5	Normal Distribution	479
10.1.5.6	Exponential Distribution	480
10.1.5.7	Gamma Distribution	481
10.1.5.8	Weibull Distribution	481
10.1.6	Applications	483
10.1.6.1	Sound Examples From Random Number Sequences	483
10.1.6.2	The Application of Random Numbers to Musical Parameters	484
10.1.6.3	Applications Using Variable Distributions	485
10.1.6.4	Choosing Among Random Values	487
10.2	Stochastic Processes	487
10.2.1	Introductory Examples and Concepts	487
10.2.1.1	Games of Chance	487
10.2.1.2	White Noise	488
10.2.1.3	General Formulation of the Concepts	489
10.2.2	Markov Chains	490
10.2.2.1	Introductory Examples	490
10.2.2.2	Definition	492
10.2.2.3	Transition Matrix and State Vectors	493
10.2.2.4	Applications	494
10.2.2.5	Markov Chains With Variable Transition Probabilities	497
10.2.2.6	Markov Chains With Variable States	499
10.2.2.7	Other Examples	499
10.2.3	More Stochastic Processes	502
10.2.3.1	Processes With Independent Increments	502
10.2.3.2	Random Walk	503
10.2.3.3	Describing a Process By Its Spectrum	505
10.2.3.4	Processes Involving Previous Events	506
10.2.3.5	Processes With Sieved Random Variables	507
10.3	Other Techniques Used for Composition.	508
10.3.1	Cellular Automata	508
10.3.1.1	One-Dimensional Automata With Two States	508
10.3.1.2	One-Dimensional Automata With Many States	510

10.3.1.3	Two-Dimensional Automata With Two States	513
10.3.2	The Golden Ratio	514
10.3.2.1	Definition and Classical Construction	515
10.3.2.2	Fibonacci Numbers	516
10.3.2.3	Continued Fractions, Surds and Golden Ratio	516
10.3.2.4	Fractals	518
10.3.2.5	A Process of Natural Growth.	518
10.3.2.6	Applications	520
10.3.3	Chaos Theory	525
10.3.3.1	Concepts	525
10.3.3.2	The Theory of Dynamic Systems	526
10.3.3.3	Self-Similarity and Fractals	527
10.3.3.4	Applications to Music	528
10.3.4	Simulating Swarm Behavior	531
10.3.4.1	The Classical Boids Algorithm	531
10.3.4.2	Extensions of the Swarm Algorithm	533
10.3.4.3	Implementations	533
10.3.5	Toward a Topology of Sounds	535
10.3.5.1	Mental Representation	535
10.3.5.2	Complex Topologies to Represent Pitch Space.	537
10.3.5.3	Time	539
10.3.5.4	Timbre	540
10.3.5.5	Position Space	541
10.3.5.6	Mapping	542
10.3.5.7	Systems	543

Appendix A Fundamentals of Mathematics 545

A.1	Numbers and Arithmetic Operations	545
A.1.1	Numbers	545
A.1.2	Rules of Algebra	545
A.2	Statements, Sets and Operations on Sets	546
A.2.1	Statements	546
A.2.2	Sets	547
A.2.3	Subsets and Power Set	547
A.2.4	Operations on Sets	547
A.2.5	The Cartesian Product	547
A.3	Equations	548
A.3.1	Definitions and Concepts	548
A.3.2	Equivalence Transformations	548
A.3.3	Algebraic Equations	548
A.3.4	Systems of Linear Equations and Matrices	549
A.3.5	Transcendental Equations	550
A.4	Functions	551
A.4.1	Definition	551

A.4.2	Properties of Functions and Graphical Representation	552
A.4.3	Basic Functions	553
A.4.4	Composite Functions.	562
A.4.5	Parametric Representation	562
A.4.6	Functions of Several Variables	563
A.4.7	Even and Odd Functions	564
A.4.8	Sequences and Series	564
A.5	Calculus	565
A.5.1	The Derivative of a Function	565
A.5.2	Rules for Differentiation	566
A.5.3	The Indefinite Integral of a Function	567
A.5.4	Integration Formulas	567
A.5.5	The Definite Integral	568
A.5.6	Partial Derivatives	568
Appendix B Tables		571
B.1	Pitches	571
B.2	Formants of English Vowels	571
B.3	Constants	572
B.4	Fibonacci Numbers	572
B.5	Prime Numbers	573
B.6	Bessel Functions	574
B.7	Chebyshev Polynomials	575
Bibliography		577
Books (by Author)		577
Books (by Number of Reference)		580
Articles from the Computer Music Journal (CMJ)		584
Articles from Other Sources		585
Articles Available on the Internet		586
Index		587