

Contents

Introduction	1
References	5
Chapter 1 Global ABCs for Second Order Elliptic Equations	9
1.1 Exterior Problem of Second Order Elliptic Equations	9
1.2 Global ABCs for the Exterior Problem of 2-D Poisson Equation.....	13
1.2.1 Steklov-Poincaré Mapping for the Exterior Problem of Laplace Equation	14
1.2.2 The Reduced Boundary Value Problem on Ω_i	17
1.2.3 Finite Element Approximation of the Reduced Boundary Value Problem (1.2.30)~(1.2.32).....	21
1.3 Global ABCs for the Exterior Problems of 3-D Poisson Equation.....	26
1.3.1 Exact and Approximate ABCs on the Spherical Artificial Boundary Γ_R	26
1.3.2 Equivalent and Approximate Boundary Value Problems on the Bounded Computational Domain Ω_i	30
1.3.3 Finite Element Approximation of the Variational Problem (1.3.30).....	34
1.4 Exterior Problem of the Modified Helmholtz Equation.....	37
1.4.1 Global Boundary Condition of the Exterior Problem for the 2-D Modified Helmholtz Equation.....	37
1.4.2 The Reduced Boundary Value Problem on the Computational Domain Ω_i	39
1.4.3 Finite Element Approximation of the Reduced Boundary Value Problem	45
1.4.4 Global Boundary Condition of the Exterior Problem for the 3-D Modified Helmholtz Equation.....	47
1.5 Global ABCs for the Exterior Problems of the Helmholtz Equation	49
1.5.1 Dirichlet to Sommerfeld Mapping of the Exterior Problem of the 2-D Helmholtz Equation.....	49
1.5.2 Dirichlet to Sommerfeld Mapping of the Exterior Problem of the 3-D Helmholtz Equation.....	55

References	58
Chapter 2 Global ABCs for the Navier System and Stokes System.....	61
2.1 Navier System and Stokes System.....	61
2.2 The Exterior Problem of the 2-D Navier System	64
2.2.1 The Global Boundary Condition on the Artificial Boundary Γ_R	65
2.2.2 The Reduced Problem on the Bounded Domain.....	71
2.2.3 The Finite Element Approximation for the Reduced Problem (2.2.59).....	77
2.3 Exterior Problem of the 2-D Stokes System.....	79
2.3.1 Highly Accurate Approximate Artificial Boundary Condition	80
2.3.2 Finite Element Approximation on the Computational Domain Ω_i for the Reduced Problem	84
2.4 Vector Fields on the Spherical Surface	91
2.5 Global ABCs for the Exterior Problem of 3-D Navier System.....	96
2.5.1 Highly Accurate Approximate ABCs	96
2.5.2 Finite Element Approximation of the Variational Problem on the Bounded Computational Domain Ω_i	100
References	111
Chapter 3 Global ABCs for Heat and Schrödinger Equations.....	115
3.1 Heat Equations on Unbounded Domains.....	115
3.2 1-D Heat Equations on Unbounded Domains.....	117
3.2.1 Exact Boundary Conditions on the Artificial Boundary Σ_0	117
3.2.2 Finite Difference Approximation for the Reduced Problem (3.2.7)~(3.2.10)	119
3.2.3 Stability Analysis of Scheme (3.2.29)~(3.2.33)	126
3.3 Global Boundary Conditions for Exterior Problems of 2-D Heat Equations	131
3.3.1 Exact and Approximate Conditions on the Artificial Boundary Σ_R	132
3.3.2 Finite Difference Approximation of the Reduced Problem (3.3.37)~(3.3.40)	138
3.4 Global Boundary Conditions for Exterior Problems of 3-D Heat Equations	140
3.4.1 Exact and Approximate Conditions on the Artificial Boundary Σ_R	140
3.4.2 Stability Analysis for the Reduced Initial Boundary Value Problem	147

3.4.3	The Finite Element Approximation for the Reduced Initial Boundary Value Problem (3.4.38)~(3.4.41)	150
3.5	Schrödinger Equation on Unbounded Domains	151
3.6	1-D Schrödinger Equation on Unbounded Domains	152
3.6.1	The Reduced Initial Value Problem and its Finite Difference Approximation	153
3.6.2	Stability and Convergence Analysis of Scheme (3.6.19)~(3.6.22)	158
3.7	The Global Boundary Condition for the Exterior Problem of the 2-D Linear Schrödinger Equation	166
3.7.1	Exact and Approximate Boundary Conditions on the Artificial Boundary Σ_R	167
3.7.2	Stability Analysis of the Reduced Approximate Initial Boundary Value Problem	172
3.8	The Global Boundary Condition for the Exterior Problem of the 3-D Linear Schrödinger Equation	175
3.8.1	Exact and Approximate Boundary Conditions on the Artificial Boundary Σ_R	176
3.8.2	Stability Analysis of the Reduced Approximate Initial Boundary Value Problem	183
	References	187

Chapter 4 ABCs for Wave Equation, Klein-Gordon Equation, and Linear KdV Equations

4.1	1-D Wave Equation	189
4.1.1	Transparent Boundary Conditions on the Artificial Boundaries Σ_1 and Σ_0	190
4.2	2-D Wave Equation	192
4.2.1	Absorbing Boundary Conditions	193
4.2.2	The Initial Boundary Value Problem on the Bounded Computational Domain D_i	200
4.3	3-D Wave Equation	203
4.3.1	Absorbing Boundary Condition on the Artificial Boundary Σ_R	204
4.3.2	The Equivalent and Approximate Initial Boundary Value Problem on the Bounded Computational Domain D_i	208
4.4	1-D Klein-Gordon Equation	209
4.4.1	Absorbing Boundary Conditions on the Artificial Boundary Σ_1 , Σ_0	210
4.4.2	The Initial Boundary Value Problem on the Bounded Computational Domain D_i	212
4.5	2- and 3-D Klein-Gordon Equations	214

4.5.1	Absorbing Boundary Conditions on the Artificial Boundary Σ_R (2-D case)	215
4.5.2	Absorbing Boundary Conditions on the Artificial Boundary Σ_R (3-D case)	220
4.5.3	The Initial Boundary Value Problem on the Bounded Computational Domain D_i	223
4.6	Linear KdV Equation	224
4.6.1	Absorbing Boundary Condition on the Artificial Boundaries Σ_a and Σ_b	225
4.6.2	The Equivalent Initial Boundary Value Problem on the Bounded Computational Domain	227
4.7	Appendix: Three Integration Formulas	228
	References	232
Chapter 5	Local Artificial Boundary Conditions	233
5.1	Local Boundary Conditions for Exterior Problems of the 2-D Poisson Equation	234
5.1.1	Local Boundary Condition on the Artificial Bboundary Γ_R	234
5.1.2	Finite Element Approximation Using the Local Boundary Condition and its Error Estimate	236
5.2	Local Boundary Conditions for the 3-D Poisson Equation	241
5.2.1	The Local Boundary Condition on the Artificial Boundary Γ_R for Problem (I)	242
5.2.2	Local Boundary Conditions on the Artificial Boundary Γ_R for Problem (II)	250
5.3	Local ABCs for Wave Equations on Unbounded Domains	254
	References	257
Chapter 6	Discrete Artificial Boundary Conditions	259
6.1	Boundary Condition on a Polygon Boundary for the 2-D Poisson Equation—The Method of Lines	260
6.1.1	Discrete Boundary Conditions on Polygonal Boundaries	260
6.1.2	Numerical Approximation of the Exterior Problem (6.1.1)~(6.1.3)	268
6.2	2-D Viscous Incompressible Flow in a Channel—Infinite Difference Method	270
6.2.1	2-D Viscous Incompressible Flow in a Channel	270
6.2.2	Discrete ABCs	272
6.3	Numerical Simulation of Infinite Elastic Foundation—Infinite Element Method	278

6.3.1	The Steklov-Poincarè on an Artificial Boundary of Line Segments	279
6.3.2	Numerical Approximation for the Bilinear Form $B(u, v)$	281
6.3.3	A Direct Method for Solving the Infinite System of Algebraic Equations (6.3.25).....	284
6.3.4	A Fast Iteration Method for Computing the Combined Stiffness Matrix K_Z	289
6.4	Discrete Absorbing Boundary Condition for the 1-D Klein-Gordon Equation— Z transform method	292
6.4.1	Z Transform.....	292
6.4.2	Discrete Absorbing ABC	294
6.4.3	Finite Difference Approximation for the 1-D Klein-Gordon Equation on the Bounded Domain.....	296
	References	297
Chapter 7	Implicit Artificial Boundary Conditions	299
7.1	Implicit Boundary Condition for the Exterior Problem of the 2-D Poisson Equation	300
7.1.1	The Single and Double Layer Potential, and Their Derivative for the 2-D Laplace Equation	300
7.1.2	The Derivation of the Implicit ABC for the Exterior Problem of the 2-D Poisson Equation.....	305
7.1.3	The Finite Element Approximation and Error Estimate for the Variational Problem (7.1.37)	309
7.2	Implicit Boundary Condition for the Exterior Problem of the 3-D Poisson Equation	310
7.3	ABC for the Exterior Problem of the Helmholtz Equation.....	316
7.3.1	The Normal Derivative on Γ_A for the Double Layer Potential of the Helmholtz Equation.....	318
7.4	Implicit ABCs for the Exterior Problems of the Navier System.....	321
7.4.1	Fundamental Solution, Stress Operator, Single and Double Layer Potentials	321
7.4.2	New Forms of $T(\partial_x, n_x)v_{II}(x)$ on Γ_A ($n = 2$)	323
7.4.3	New Forms of $T(\partial_x, n_x)v_{II}(x)$ on Γ_A ($n = 3$)	328
7.4.4	Implicit ABC for the Exterior Problem	333
7.5	Implicit ABCs for the Sound Wave Equation.....	336
7.5.1	The Kirchhoff Formula for the 3-D Sound Wave Equation	337
	References	338

Chapter 8 Nonlinear Artificial Boundary Conditions	341
8.1 The Burgers Equation	342
8.1.1 Nonlinear ABCs for the Burgers Equation.....	343
8.1.2 The Equivalent Initial Boundary Value Problem on the Bounded Computational Domain D_i	346
8.2 The Kardar-Parisi-Zhang Equation	348
8.2.1 Nonlinear ABC for the K-P-Z Equation ($D = 1$).....	349
8.2.2 Nonlinear ABC for the K-P-Z Equation ($D = 2$).....	350
8.2.3 Nonlinear ABC for the K-P-Z Equation ($D = 3$).....	353
8.3 The Cubic Nonlinear Schrödinger Equation.....	354
8.3.1 Nonlinear Boundary Conditions on the Artificial Boundaries Σ_0 and Σ_{-1}	355
8.3.2 The Equivalent Initial Boundary Value Problem on the Bounded Domain $[-1, 0] \times [0, T]$	356
8.4 Operator Splitting Method for Constructing Approximate Nonlinear ABCs	358
8.4.1 The Local Absorbing ABC for the Linear Schrödinger Equation	359
8.4.2 Finite Difference Approximation on the Bounded Computational Domain	360
References	362
 Chapter 9 Applications to Problems with Singularity	365
9.1 The Modified Helmholtz Equation with a Singularity	366
9.1.1 ABC Near Singular Points	367
9.1.2 An Iteration Method Based on the ABC	368
9.2 The Interface Problem with a Singularity	373
9.2.1 A Discrete Boundary Condition on the Artificial Boundary Γ_R	374
9.2.2 Finite Element Approximation.....	379
9.3 The Linear Elastic Problem with a Singularity	380
9.3.1 Discrete Boundary Condition on the Artificial Boundary Γ_R	382
9.3.2 An Iteration Method Based on the ABC	390
9.4 The Stokes Equations with a Singularity	393
9.4.1 The Discrete Boundary Condition on the Artificial Boundary Γ_R	394
9.4.2 Singular Finite Element Approximation.....	403
References	406
 Bibliography	409