Contents

Part I Stochastic Dependence and Extremal Risk

1	Сорі	ılas, Sklar	r's Theorem, and Distributional Transform	3
	1.1	Sklar's T	Theorem and the Distributional Transform	3
	1.2	Copula I	Models and Copula Constructions	7
		1.2.1	Some Classes of Copulas	8
		1.2.2	Copulas and L^2 -Projections	11
	1.3	Multivar	riate Distributional and Quantile Transform	14
	1.4	Pair Cop	oula Construction of Copula Models	17
	1.5 Applications of the Distributional Transform		tions of the Distributional Transform	21
		1.5.1	Application to Stochastic Ordering	21
		1.5.2	Optimal Couplings	23
		1.5.3	Identification and Goodness of Fit Tests	25
		1.5.4	Empirical Copula Process and Empirical	
			Dependence Function	26
	1.6	Multiva	riate and Overlapping Marginals	28
		1.6.1	Generalized Fréchet Class	28
		1.6.2	Copulas with Given Independence Structure	31
		1.6.3	Copulas, Overlapping Marginals, and L^2 -Projections	33
2	Fréchet Classes, Risk Bounds, and Duality Theory			35
	2.1	Dual Re	presentation of Generalized Fréchet Bounds	37
	2.2	Fréchet	Bounds Comonotonicity and Extremal Risk	45
3	Convex Order, Excess of Loss, and Comonotonicity			53
	3.1	Convex	Order and Comonotonicity	53
	3.2	Schur O	Order and Rearrangements	57
	3.3	Rearran	gements and Excess of Loss	63
	3.4	Integral	Orders and $\prec_{\mathcal{F}}$ -Diffusions	66



ix

4	Boun	ids for the Distribution Function and Value at Risk of	
	the J	oint Portfolio	71
	4.1	Standard Bounds	72
	4.2	Conditional Moment Method	79
	4.3	Dual Bounds	82
5	Resti	rictions on the Dependence Structure	91
	5.1	Restriction to Positive Dependent Risk Vectors	91
	5.2	Higher Order Marginals.	95
		5.2.1 A Reduction Principle and Bonferroni Type Bounds	97
		5.2.2 The Conditioning Method	103
		5.2.3 Reduction Bounds for the Joint Portfolio in	
		General Marginal Systems	107
6	Depe	endence Orderings of Risk Vectors	
	and l	Portfolios	113
	6.1	Positive Orthant Dependence and Supermodular Ordering	113
	6.2	Association, Conditional Increasing Vectors, and	
		Positive Supermodular Dependence	120
	6.3	Directionally Convex Order	124
		6.3.1 Basic Properties of the Directionally Convex Order	124
		6.3.2 Further Criteria for \leq_{dex}	126
		6.3.3 Directionally Convex Order in Functional Models	128
	6.4	Dependence Orderings in Models with Multivariate Marginals	131
'n			
Pa	rt II	Risk Measures and Worst Case	

Portfolios

7	Risk Measures for Real Risks			141
	7.1	Some	Classes of Risk Measures for Real Variables	142
		7.1.1	Basic Properties of Risk Measures	142
		7.1.2	Examples of Risk Measures	146
	7.2	Repres	entation and Continuity Properties of Convex	
		Risk M	feasures on L ^p -Spaces	153
		7.2.1	Convex Duality and Continuity Results	154
		7.2.2	Representation of Coherent and Convex Risk	
			Measures on L^p	157
		7.2.3	Continuity Results for Risk Measures on L^p	160
8	Risk Measures for Portfolio Vectors		167	
	8.1	Basic l	Properties of Portfolio Risk Measures	168
	8.2	Classe	s of Examples of Portfolio Risk Measures	174
		8.2.1	Aggregation Type Risk Measures	174
		8.2.2	• Multivariate Distortion and Quantile-Type	
			Risk Measures	180

	8.3	Representation and Continuity of Convex Risk	
		Measures on L_d^p	184
9	Law	Invariant Convex Risk Measures on L_d^p and Optimal	
	Mass	s Transportation	189
	9.1	Law Invariant Risk Measures and Optimal Mass	
		Transportation	190
	9.2	Multivariate Comonotonicity and the <i>n</i> -Coupling Problem	198
	9.3	Worst Case Portfolio Vectors and Diversification Effects	207
	9.4	Examples of Worst Case Risk Portfolios and Worst	
		Case Diversification Effects	214

Part III Optimal Risk Allocation

10	Optir	nal Allocations and Pareto Equilibrium	227
	10.1	Pareto Equilibrium and Related Risk Measures	
		in the Coherent Case	227
	10.2	Optimal Allocations Under Admissibility Restrictions	235
	10.3	Pareto Equilibrium for Convex Risk Measures	248
	10.4	Pareto Optimality, Comonotonicity, and Existence	
		of Optimal Allocations	256
11	Char	acterization and Examples of Optimal Risk	
	Alloc	ations for Convex Risk Functionals	265
	11.1	Inf-Convolution and Convex Conjugates	266
	11.2	Characterization of Optimal Allocations	269
	11.3	Examples of Optimal Risk Allocations	276
		11.3.1 Expected Risk Functionals	277
		11.3.2 Dilated Risk Functionals	278
		11.3.3 Average Value at Risk and Stop-Loss Contracts	279
		11.3.4 Mean Variance Versus Standard Deviation	
		Risk Functionals	280
	11.4	Optimal Allocation of Risk Vectors	283
		11.4.1 Characterization of Optimal Allocations	284
		11.4.2 Law Invariant Risk Measures and Comonotonicity	289
		11.4.3 Existence of Minimal Risk Allocations	293
		11.4.4 Uniqueness of Optimal Allocations	299
		11.4.5 Examples of Optimal Allocations	300
	11.5	The Capital Allocation Problem	302
12	Opti	mal Contingent Claims and (Re)insurance Contracts	305
	12.1	Optimal Contingent Claims	305
		12.1.1 Optimal Investment Problems	306
		12.1.2 Minimal Demand Problem	309
	12.2	Optimal (Re)insurance Contracts	314
		12.2.1 Optimality of Stop-Loss Contracts	314
		12.2.2 Optimal Worst Case (Re)insurance Contracts	318

Part IV Optimal Portfolios and Extreme Risks

13	Optir	nal Portfolio Diversification w.r.t. Extreme Risks	325
	13.1	Heavy-Tailed Portfolios and Multivariate Regular Variation	325
	13.2	Extreme Risk Index and Portfolio Diversification	328
	13.3	Estimation of the Extreme Risk Index and the Optimal	
		Portfolio	333
	13.4	Asymptotic Normality of $\hat{\gamma}_{\xi}$	342
	13.5	Application to Risk Minimization	350
14	Orde	ring of Multivariate Risk Models with Respect to	
	Extre	eme Portfolio Losses	353
	14.1	Asymptotic Portfolio Loss Ordering	353
	14.2	Characterization of \leq_{anl} in Multivariate Regularly	
		Varying Models	359
		14.2.1 Multivariate Regular Variation	359
		14.2.2 Ordering of Canonical Spectral Measures	365
		14.2.3 Unbalanced Tails	372
	14.3	Relations to the Convex and Supermodular Order	374
	14.4	Examples of apl-Ordering	379
Ref	erence	S	385
Lis	t of Sy	mbols	399
Ind	ex		401